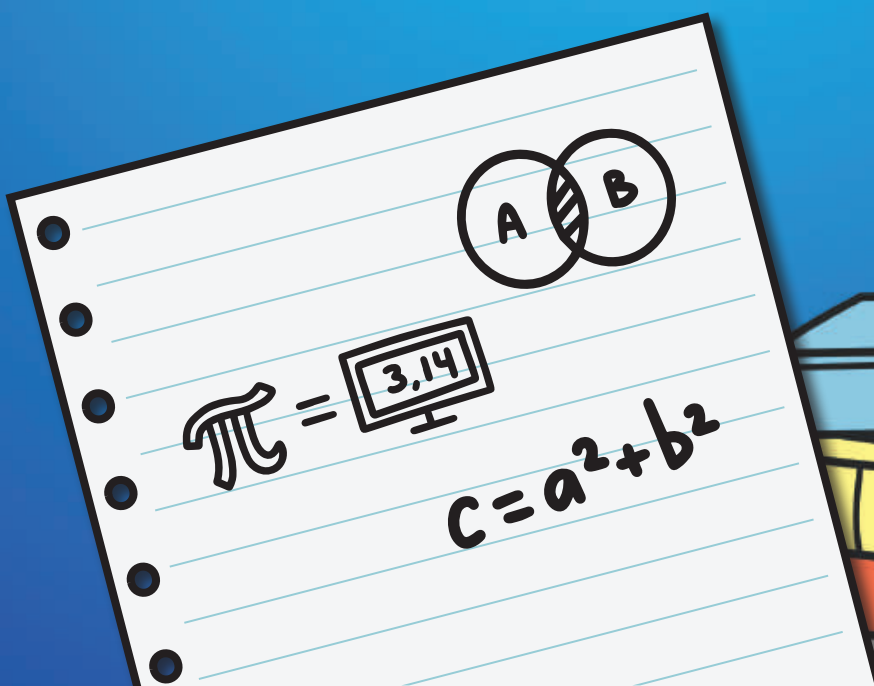




TalkforWriting

How Talk for Writing supports Maths



2 HOW TALK FOR WRITING SUPPORTS MATHS

For schools already using the Talk for Writing (TfW) approach in English, integrating the key elements of TfW into how you teach maths, alongside all the other subjects, gives the children and the teachers one unifying, powerful approach to learning across the curriculum.

This resource and all accompanying videos can be found at:
<http://www.talk4writing.com/maths>

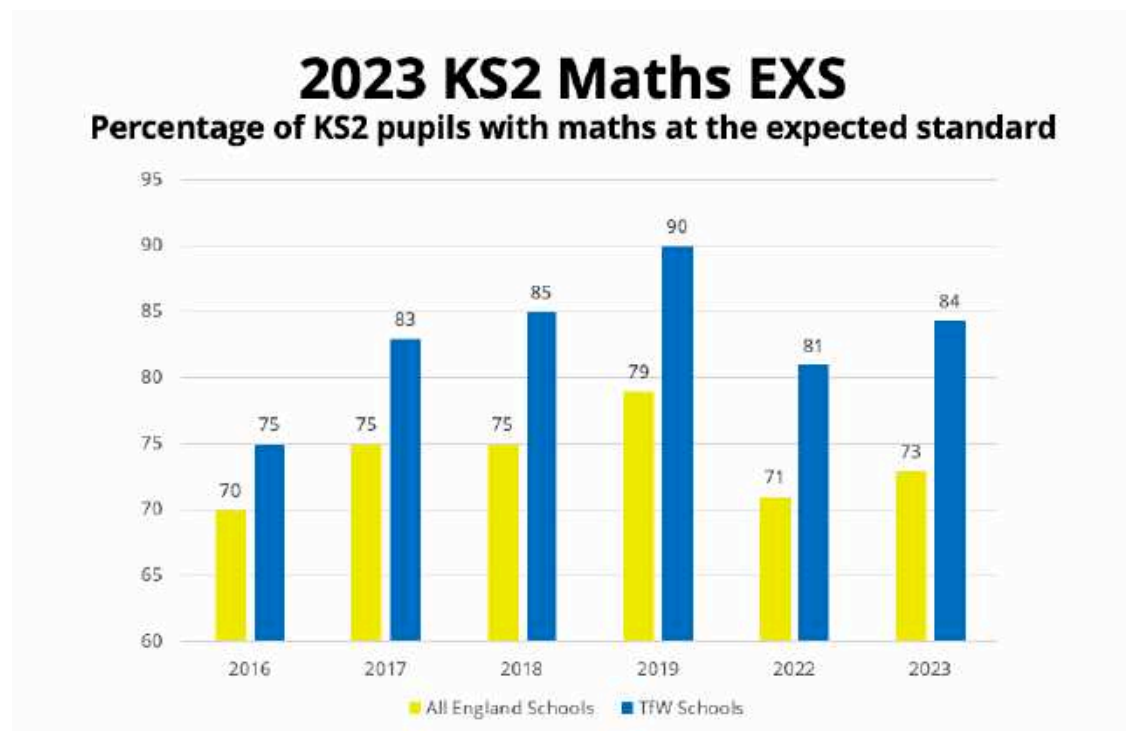
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Introduction

Julia Strong and Pie Corbett

How Talk for Writing Supports Maths is very much still work in progress. Therefore, we are providing this free online resource for schools within the Talk for Writing (TfW) network so that together we can build a more powerful understanding of how TfW supports maths. We very much hope that you will use this resource and feedback your findings to Julia Strong (Julia.strong@talk4writing.com).

You only have to look at the graph below, comparing maths results in TfW training schools with the results in all primary schools in England, to see that the TfW approach makes a real difference to maths, particularly considering that the intake of most TfW schools is very challenging.



This resource is based on the work of Talk for Writing schools that have developed how the approach supports maths. Particular thanks go to Tracey Adams, Deputy Headteacher and Maths Lead at St Matthew's C of E Primary School, Birmingham, (now Headteacher at Christchurch C of E Primary School, Birmingham) and Nick Warren, Year 6 teacher and maths lead at Briar Hill Primary School, Northampton, without whom this resource would not exist. See **pages 44-46** to find out why both schools use TfW to support maths.

Schools select the maths scheme that they think works best for their children: St Matthew's, for example, uses the *White Rose* scheme, while Briar Hill uses *Effective Maths*. This resource shows how TfW can augment the effectiveness of such schemes by supporting children in talking their way to understanding. Tracey and Nick have chosen to illustrate this by focusing on one strand of the maths curriculum, fractions.

4 HOW TALK FOR WRITING SUPPORTS MATHS

This resource is a form of teacher's notes to support any school already using the TFW approach in English so that they can provide CPD for their school. It includes 11 film clips. Watching film in training sessions is usually more effective if the audience is given a question to consider while watching the clip. Possible questions have been integrated into the text to support in-house training.

To see the process in action, and for information about training, visit <https://www.talk4writing.com/training-centres/>

Set a cold task to establish the baseline and guide planning

Cold tasks are important in maths units, just as they are in English, because every class is different. In order to create a sequential journey for the teaching of maths units, teachers create a cold task to share with their children, including questions linked to each objective. This is used to gather intelligence and to answer two key questions.

1. What are the children confident with? (This can then be a focus for retrieval practice within the unit.)
2. What will need to be explicitly taught because the children are less confident in these areas?

Both Nick and Tracey emphasise how essential the cold task is to effective maths teaching:

“Talk for Writing’s cold-to-hot process has enabled staff to have a clear understanding of formative baseline assessment and also provides a great summative assessment tool. This allows teachers to focus on developing the areas that most need support while building on the children’s prior knowledge.”

Nick Warren

“Prior to this joined-up way of thinking, teachers did not have a clear understanding of what children were able to do within a specific area of maths. The teachers lacked a clear way to assess what children were bringing to the table and what they were able to do after a unit of work had been taught. Talk for Writing’s cold-to-hot-task process helped to bring real clarity to the data teachers had before beginning to teach and gave them the knowledge needed to develop connections as the teaching continued, alongside building the children’s mathematical skills and hence boosting confidence.”

Tracey Adams

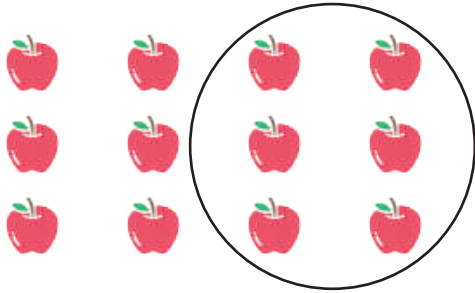
On the next page is the cold task Tracey used to establish whether her Year 2 children understood simple fractions alongside understanding the equivalence of $\frac{1}{2}$ to $\frac{2}{4}$ to $\frac{3}{6}$ and recognising, naming and writing fractions ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{4}$ and $\frac{3}{4}$ of a length, a shape, a set of objects or quantity).

Tracey warmed this up by reminding the children of when they’d done fractions before and would read the questions to the children, explaining any vocabulary they might not be sure of.

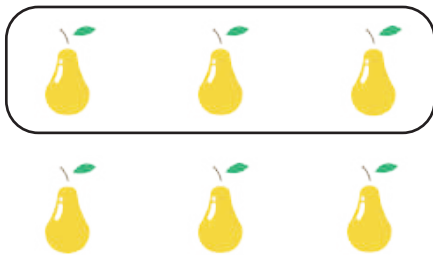
6 HOW TALK FOR WRITING SUPPORTS MATHS

1. $\frac{1}{4}$ of 20 is ____ (**15** or **10** or **4** or **5**).
2. Which statement does not match the image?

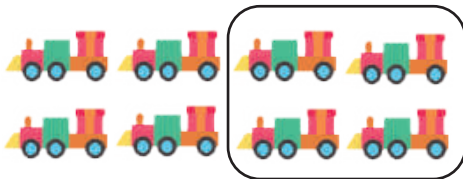
a. $\frac{3}{4}$ of the apples



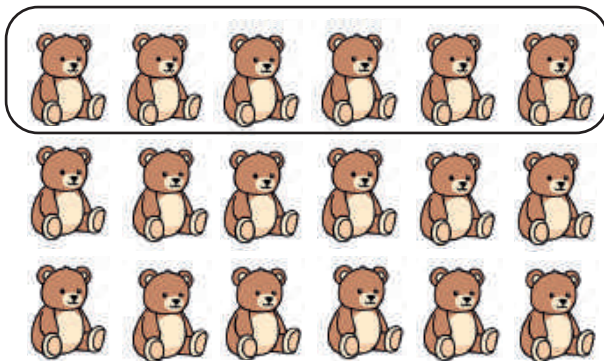
b. $\frac{1}{2}$ of the pears



c. $\frac{3}{4}$ of the trains



d. $\frac{1}{2}$ of the teddies



3. $\frac{1}{2}$ is equal to ____ .
 - a. $\frac{4}{4}$
 - b. $\frac{3}{4}$
 - c. $\frac{1}{4}$
 - d. $\frac{2}{4}$

4. What fraction of the balloons are green?



- a. $\frac{1}{2}$
- b. $\frac{3}{4}$
- c. $\frac{1}{4}$
- d. $\frac{3}{4}$

The images on these two pages are based on an example from White Rose Maths' diagnostic questions (Whiterosemaths.com).
© 2023 White Rose Education

Tracey's advice is that the teacher must be clear when they put the questions together that the children have understood the objectives covered in the unit – have they understood what they have been learning? So, in this case, do they understand the difference between a denominator and a numerator? Do they understand the symbolism of a fraction? Do they understand that to grasp what the fraction is you have to know what the whole is? Looking at the children's cold tasks enables the teacher to see any misconceptions the children have relating to these key aspects of fractions.

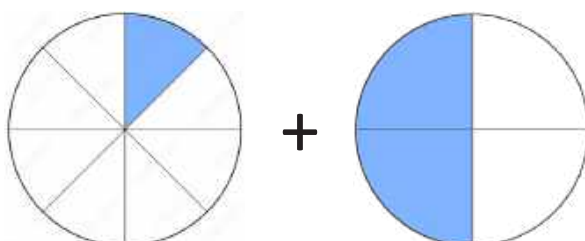
What a fractions cold task looks like by Year 6

At Briar Hill, fractions in Year 6 are split into two units, and there is also a unit on decimals and money, percentages and ratios in line with National Curriculum guidance. For an overview of the first unit, see **Appendix 4 (page 50)**.

The Year 6 cold task for the first unit on fractions is based on what they (we hope) learnt in Year 5. Nick often uses a hook just before the cold task such as looking at fractions in the real world (see **page 10**) to warm the topic up and help the children recall what they already know. Nick uses this cold task to understand what prior knowledge the children have and whether the concepts are secure. The questions involve several arithmetic and reasoning aspects.

Typical misconceptions arising from this cold task are in the boxes.

1. True or False? $\frac{1}{8} + \frac{1}{2} = \frac{5}{8}$



Typical misconception: Answer = $\frac{3}{12}$ as there are three shaded parts and 12 parts altogether.

8 HOW TALK FOR WRITING SUPPORTS MATHS

2a. $\frac{3}{8} + \frac{1}{4} =$

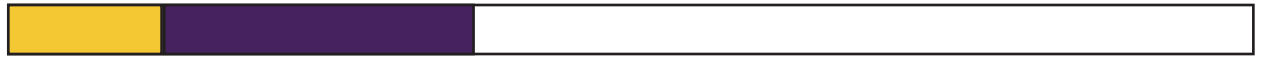
2b. $1\frac{1}{4} + \frac{5}{8} =$

Typical misconceptions:

2a. Answer = $\frac{4}{8}$

2b: Answer $1\frac{1}{4} + \frac{5}{8} = \frac{27}{8}$

3. $\frac{1}{8}$ of the rectangle below is shaded in yellow while $\frac{1}{4}$ of the rectangle is shaded in purple. What fraction of the rectangle is not shaded?

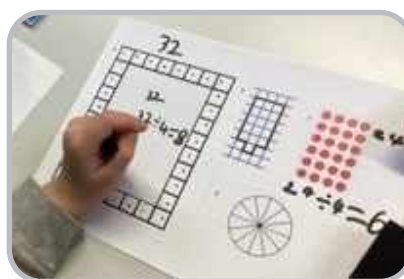


Typical misconception: $\frac{3}{8}$

If these misconceptions were typical of the class, the teacher would know from questions 1 and 2a that there is a general failure to understand denominators or equivalent fractions. The question 2b misconception would show that many children mistook $1\frac{1}{4}$ for $\frac{1}{4}$. They would need to be helped to understand that the whole is $\frac{1}{4}$ (4 divided by 4 = 1). Was this a visual misconception? The question 3 misconception suggests that they have misread the question.

If the earlier teaching has been sound, such misconceptions would not be widespread but would only apply to individuals/small groups who haven't grasped key underpinning concepts. Specific scaffolding would need to be planned for these groups to help them grasp these key concepts. This would probably be best achieved through concrete resources, getting them to understand physically what the fractions represent. If an individual child cannot grasp a key concept, you may have to go right back to where their understanding first fails and build all the stages. Without this understanding, they will not be able to progress. For example, if a child in Year 4 can't do simple times tables, you must go all the way back to number bonds. You must go right back so that they understand the concept of number.

Concrete resources are essential to help children understand mathematical concepts. The pictures here from Briar Hill demonstrate how supportive such resources are.



The additional questions in the Year 6 cold task are these:

4. True or False? $\frac{5}{6} - \frac{1}{3} = \frac{4}{6}$

5a. $\frac{2}{3} - \frac{1}{6} =$

5b. $\frac{1}{3} - \frac{1}{12} =$

6a. $2\frac{1}{2} - \frac{1}{5} =$

6b. $1\frac{1}{10} - \frac{2}{5} =$

7. Nicole has **12** bouncy balls in her collection. She gives her friends **two thirds** of her collection. How many does she have left?

Nick explained that the cold tasks show which concepts have been embedded but also if other concepts need reviewing, e.g. sixths and eighths. The first session would then be a review based on findings from the cold task: understanding what gaps need filling. These tasks also help the teacher see if the children have developed other skills in different maths units that link into fraction skills, like division and multiplication, so they now know how to divide by 10, 100 and 1000, which is going to help them understand when doing percentages. Cold tasks also enable the teacher to review the room, seeing what methods are used, who is needing concrete resources and/or what concepts are secure and fully embedded (or not).

Given the sequential way in which maths is taught, so that each key area is returned to at least every year, it can be very useful to set the hot task from the last time the class maths unit focused on, say, fractions as the cold task for the next time they begin a unit on fractions.

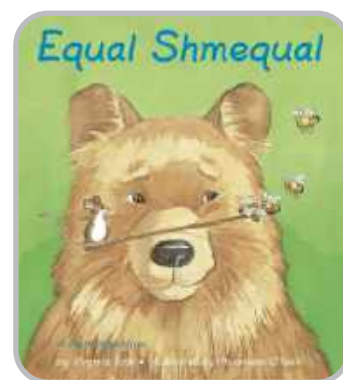
Chapter 1: The Imitation Stage

Using TFW strategies to build engagement, understanding and retention

1. Begin with a hook

The more we can motivate children to be interested in maths and find everyday mathematical links that they can relate to, the more likely they are to engage with mathematical problems. At the beginning of each maths unit, teachers plan a hook. Simple examples showing maths in action in everyday life often work best.

Tracey points out that stories can provide a great opportunity to hook children in to the application of mathematical technical terms. For example, in the story pictured here, the terms *equal* and *unequal* are central to understanding the story. Mouse has created a seesaw and then engages with her friends to make both sides of the seesaw equal.



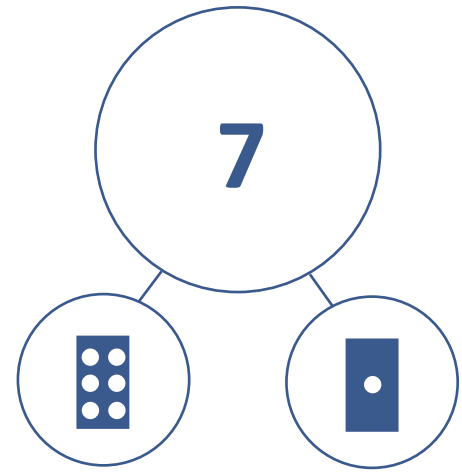
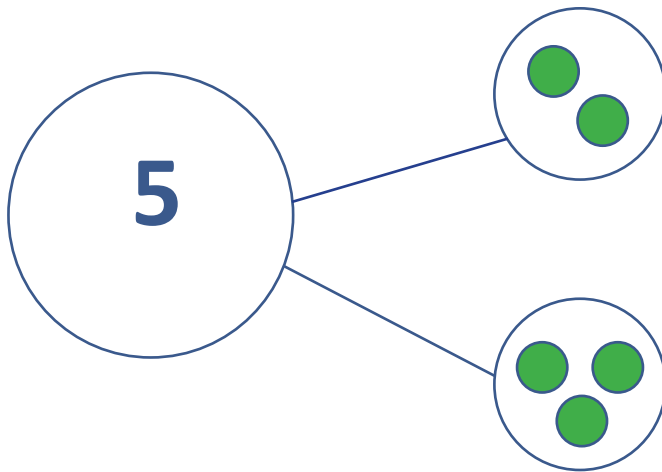
It's always important to show the children how the maths they are about to learn is useful in everyday life. Tracey gave the practical example of referring to a clock and mentioning that the school day ends at 3.00 – which is a quarter of the way round the circle of a clock face. The children like these sorts of examples.

Nick gave this practical example. At Briar Hill, they often give pens to Year 6 classes for English as a reward. Nick decided to turn the upcoming pens award into a mathematical hook at the beginning of his fractions unit so he said to his class: **“Here is a pack of 50 pens. If every child in this class of 30 children is to receive a pen, what percentage will be left? And how can we express that percentage as a fraction?”** It engaged the class and stuck in their heads.

2. Introduce and embed key vocabulary

When planning any unit, it is essential to identify the key vocabulary that will underpin the children's understanding of the concepts being taught. For her Year 2 fractions unit, Tracey identified these 10 key words/terms: *whole, parts, equal, numerator, denominator, equivalence, sharing, grouping, unit/non-unit, fraction*.

One very useful resource here to help make abstract concepts concrete is to have a part/part-whole model with counters or dominoes, as illustrated on the next page.



$$7 + 0 = 7$$

$$6 + 1 = 7$$

$$4 + 3 = 7$$

$$0 + 7 = 7$$

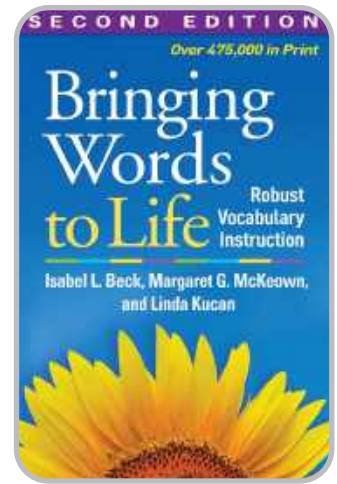
$$1 + 6 = 7$$

One TfW approach that has proved very powerful right across the curriculum is to introduce at the very beginning of a unit the words that will be key to understanding that unit, using a 'Never-heard-the-word grid' like the one below for fractions in Year 2. These can also be used as cold tasks to establish which words to focus on most to develop the children's understanding. The idea is to give every child a copy of the grid and make it clear that it is not a test – just a way of establishing which words they know and which they are not so familiar with at the start of a unit. However, point out that you will return to this at the end of the unit and expect everyone to know every word confidently. Say each word and put it into a sentence that provides context but doesn't explain its meaning. Then ask the children to tick the appropriate column ('Never heard' or 'Heard – not sure of meaning') or, if they know what it means, jot down its meaning or draw an image or a symbol to represent it. Ask the children to mark their own work and then take it in so you can spot any misconceptions and establish which words will need teaching most. Don't try to go over the meaning of the words immediately but teach them in context as they arise during the unit. If you start going over the meanings of the whole list, teachers tell me that most children will stop listening round about word three, even if they manage to look as if they are listening!

Never-heard-the-word grid for Year 2 fractions

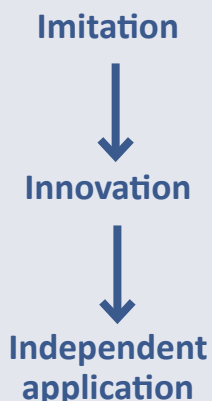
Key words	Never heard	Heard – not sure of meaning	Know what it means – jot down meaning/image/symbol
1. whole			
2. parts			
3. equal			
4. fraction			
5. numerator			
6. denominator			
7. equivalence			
8. grouping			
9. unit/non-unit			
10. sharing			

A tried-and-tested method to introduce new key words effectively is the *Isabel Beck routine*. If you are not familiar with Isabel Beck's excellent advice on how to build children's vocabulary, get hold of a copy of *Bringing Words to Life*. Her basic routine is illustrated here (note that this example assumes the children already know what *fraction* means – if not, the definition used here will only make matters worse!).



Teaching vocabulary in context The Isabel Beck routine

1. Say the word aloud:
denominator (and show how it is represented: $\frac{1}{2}$, $\frac{1}{4}$).
2. Give a **simple child-friendly** definition: *The denominator is the bottom number in any fraction.*
3. Involve children in saying the word and the definition.
4. Use the word in sentences.
5. Help children to use the word several times in different mathematical contexts.



The teacher says the word being focused on – *denominator* – and gives a simple, child-friendly definition: *The denominator is the bottom number in any fraction.* The teacher involves the children by saying the word and the definition, and then just saying the definition and asking the children to say the word. Then the teacher says the word and asks the children to say the definition. Finally, the teacher asks the children to use the word in sentences. (Given the nature of this particular word and the age of the children, this will probably have to be only in a mathematical context.) The teacher would then ask the class to use the word relating to a range of fractions.

As the image here illustrates, this process over time takes you from imitation to real independent application. The more the

word is used within the unit, the more the children will gain confidence in using it correctly and understand the word when others use it.

Enhance understanding of words by using actions

The power of actions in helping children both to remember words and to relish their meaning will be well known to any teachers familiar with the TfW method.

Interestingly, if you have to devise an image to represent the word, you have to think about its meaning. If you are asked only to write it down, this tests your spelling – it does not relate to meaning and understanding. Adding actions to images is even more effective, which is one of the reasons why the TfW approach is so powerful.

Video clip 1 from Yew Tree Community Primary School shows how actions help children remember the meaning of the key terminology relating to angles. This screenshot from the clip shows how the girl is using arm movements to indicate what an *acute angle* is, so you

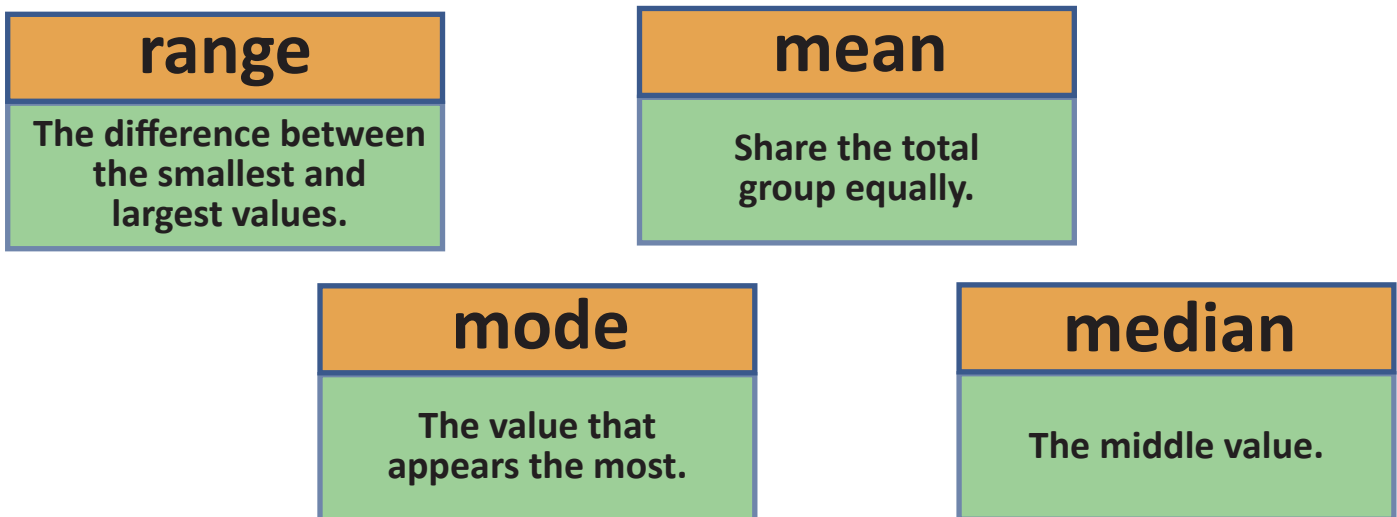


Pupil demonstrating an acute angle

know she has understood the word. And if you hold your arm upwards at an angle of 45 degrees to illustrate the *hypotenuse* side of a triangle, you will remember what 'hypotenuse' means. If you use this clip for training, you may want to ask your audience to consider this question as they watch: **“How do we know the text map and actions method has helped this girl recall which angle is which?”** Feedback might include:

- the actions have helped her understand the different types of triangles so she can now remember them;
- because she is no longer confused by which angle is which, she can refer to them confidently and explain what she is learning;
- it appeals to her competitive side as she thinks she can now win!

Video clip 2, from Knowle Park Primary, Bristol, shows that this method can even help you remember the difference between **median**, **mean**, **mode** and **range**. The actions help the children to recall visually the meaning of the words and to embed the learning in the long-term memory.



Displays can be used to strengthen understanding. The terms **mean**, **median**, **mode** and **range** tend to lead to confusion. You think you've grasped them, then the clarity fades and you sink back into confusion. A display showing the definitions of these terms helps reinforce understanding. And it helps the teacher too!

Building up vocabulary progressively

The children's comprehension of the technical language of mathematics (the Tier 3 words) will need to be introduced and embedded progressively in line with how each school delivers the maths curriculum. See **Appendix 2 (pages 47-48)** for an example of **vocabulary progression for maths** from Briar Hill.

Making the words visible

The more the children see the words you are focusing on, the more they will become familiar with them, so it is very useful to display the words alongside the images and actions that represents them, like the maths image on the right which is recreated from a display at Front Lawn Primary School, Havant. They are an excellent way of helping children remember that a number of mathematical terms can mean similar things. Pictures of the children demonstrating the actions that help recall the words' meanings of are an added bonus.

Matthew Squires, from Inglewood Junior School in Cumbria, decided to give the Tfw approach to learning key maths terminology a go as his gap task, following the first part of an online course. He found it worked very nicely and was kind enough to send us **video clip 3**. Matthew commented that doing this as part of the fractions unit had been fun, whereas often, in other years, the children had had difficulty with fractions. Matthew decided to have a go at focusing on the language of fractions to prevent it from being a barrier. His class rehearsed the language and played with it and co-constructed working walls to help recall the meanings of words. Using pictures, actions and concrete examples, alongside helping the children talk the language, has made a difference.



make

add

sum

altogether

+



minus

subtraction

subtract

-



lots of

multiplication

multiply

times

X

To find out more about the power of co-constructing working walls, see **pages 21-23**.

"I really feel this has helped develop their understanding, and I know this because of the way they are explaining fractions to each other and to me; it's been fantastic. It's just a start and I'm really excited to see where I can go with it next."

Matthew Squires, Inglewood Junior School



3. Warm up the key skills (tools) and language patterns

Helping the children talk their way to understanding is central to successful maths teaching. Throughout the **Imitation Stage**, it is essential that the children are supported in understanding each mathematical concept and procedure that is being modelled. Ensuring the children can talk their way to understanding each part of the stage is crucial. If these secure foundations are not laid, moving on to the **Innovation Stage** will be meaningless and will just add to a sense of confusion and failure.

“Aiming high while scaffolding down through talk partners is an embedded and sustaining strategy which allows children to understand concepts and internalise them.”

Nick Warren



In written subjects like English, the key vocabulary and recyclable phrasing of whatever type of text is being focused on is warmed up throughout the **Imitation Stage**, so that the children not only understand the words but can also use the key phrasing that underpins the unit. This involves modelling **recyclable generic linking phrases** (e.g. *because*) and **recyclable topic-specific phrasing** (e.g. *In order to solve the problem...*) that are liable to be used again in similar texts or discussions. This enables the children to talk the tune of the text because they now automatically use such phrases when they discuss, explain or write similar material.

This acts as a form of thinking/talking frame, which can later be innovated on and applied independently. In such a way, children internalise the language patterns (the tune) of whatever type of text or subject they are focusing on and can use them with confidence.

This process in many ways is even more important in maths, since the children have to understand the logic of what they are doing to enable them to apply their maths independently. They have to be able to explain coherently the underpinning concepts and procedures they are using so that, when faced with a maths problem, they can select which procedure to follow and know how the related concepts work.

Short-burst calculations

In English, short-burst writing throughout the **Imitation Stage** breaks down the writing techniques illustrated by the model text into small digestible skills (tools) that are modelled, discussed and practised. This is the key to building up the skills of effective writing.

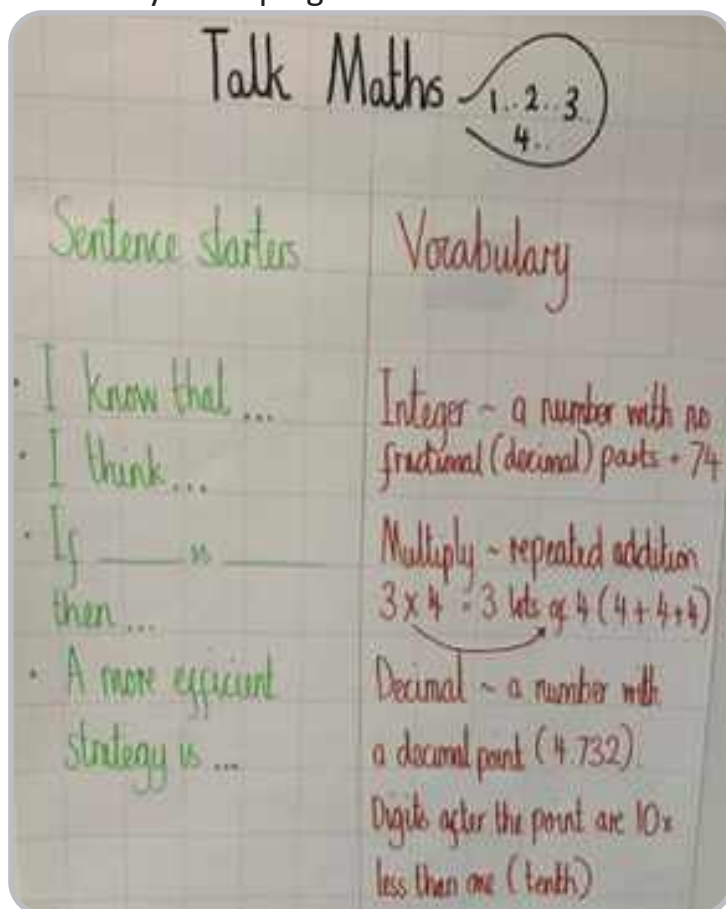
In maths, this process is mirrored not by writing but by short-burst calculations. Throughout the **Imitation Stage**, the mathematical process is broken down into small digestible skills that are modelled, discussed and practised. This is key to building up confidence in the skills that are needed to solve mathematical problems. Paired discussion of each skill underpins the teaching of all subjects. In Tracey Adams' words:

Nick Warren agreed and added:

“Short-burst calculations enable the teacher to introduce procedural variation. Once the underpinning concepts are clear, the teacher can model procedural variation so the children can select the method to solve the problem. This lies at the heart of mathematical fluency.”

“Talk for Writing techniques such as shared writes, which in maths often becomes shared calculations, co-constructing toolkits and boxing-up the order in which to do things, mean that every child is able to achieve; learning is deep and sustainable as the toolkit can be referred to and helps address any misconceptions. Not only does a toolkit or shared calculations build confidence, it also helps to ensure that learning is built on something that has already been sufficiently mastered. The children talk their way to understanding so they can reason about concepts and make connections which, in turn, develops conceptual and procedural fluency. This enables the teachers in maths to unpick what a mathematical problem is and show the thinking.”

Tracey has found talk frames to be a very powerful way of helping children understand and use the **key recyclable language of maths**, an idea that is promoted in Ofsted’s research paper on teaching the maths curriculum (<https://www.gov.uk/government/publications/research-review-series-mathematics>). To support children’s understanding of key terms and to improve the quality of responses, Tracey uses a talk maths frame related to whatever the maths focus is. The example here includes two columns: one is a set of **sentence stems** (the recyclable introductory phrases that will support the children’s ability to explain and reason in maths – and any other subjects that require logical thinking); the other column is the **key vocabulary** needed, along with definitions, to ensure children’s responses are precise and focused.



She feels that talk frames have revolutionised her teaching. They enable her to prompt children to expand on their initial one-word responses and provide a scaffold to challenge the children’s thinking. This helps the children structure their responses in a reasoned way.

In Tracey's words:

"Now, if I say, 'Tell me about the digit 4 in 4.346,' I would expect children to respond like this with the support of the frame: 'I know that one of the 4s is worth 4 hundredths because it is in the hundredths' column, after the decimal point. I think the other 4 is worth more because it is before the decimal point, in the ones' column.'"



The phrasing they need is also displayed on posters, like the Year 1 example here. Children's conceptual understanding increases alongside their confidence.

Providing the children with the **stem sentences** (the recyclable phrases that enable you to explain what you are learning) is key to helping the children talk their way to understanding. **Video clip 4** shows this in action at Briar Hill, with Year 2 teacher Hollie Tranter co-constructing with her class how to use the stem sentences to explain their understanding. The clip shows the class in pairs discussing what to do. If you are using this clip for training, you might want to ask your audience to reflect on this question while watching: **"How do the stem sentences support the children's understanding?"**

If the children are to be able to talk their way to understanding, the teacher needs to model very clearly and precisely the language the children need to explain the maths they are learning. In maths, the oral explanation of what to do and why you are doing it often replaces the model written text that would be used in English. The way the teacher explains what the class is learning, and involves the children in being able to explain this, becomes the model, as the transcript of **video clip 5** powerfully demonstrates on the next page. If you use this clip for training, you may want to ask your audience: **"How is the teacher helping his Y6 class have confidence with maths problems?"** A useful additional question is: **"Which of the phrases used by the teacher are particularly handy?"**

As you read it, watch it and hear it, you can see how Allan Crozier, using the *Effective Maths* programme at Briar Hill, has carefully planned this *think aloud/my turn/your turn* technique to guide the children through the hidden thought-processes that underpin the mathematical understanding, just as in English a think-aloud helps children understand the hidden thoughts that guide writing. The children then have the vocabulary and phrasing to enable them to understand and express their understanding. You will notice that apart from repeating all the key phrases, Allan also includes the request to **"speak in full mathematical sentences"** as well as asking for **"really clear, explicit maths vocabulary"**. He is very aware that he needs to teach the tune of maths, that the everyday word *difference* has a very specific meaning in maths. The children may never actually write this understanding down,

but they need the understanding in their heads so they can put it into practice when solving maths problems; getting children to write down their mathematical understanding has also proved powerful, as explained on **pages 38-39**.

- Teacher:** We need **really clear, explicit maths vocabulary in order to solve this problem**. So, talk to your partner really quickly and see if you can get some of that really clear, explicit vocabulary across. Off you go.
- Class:** (Discuss in pairs.)
- Teacher:** So, we've got two numbers here: minus 5 and zero. What would my first step be? What would my first step be? Chelsea?
- Chelsea:** **Find out the difference.**
- Teacher:** Excellent. So, the word that we are looking for is **difference**.
- Class:** **Difference.**
- Teacher:** **Difference.**
- Class:** **Difference.**
- Teacher:** So, we're **finding the difference**. Well done, Chelsea.
- Teacher:** Who can tell me what the **difference** is between these two numbers? **Let's have it in a full mathematical sentence, please.** Jenson?
- Jenson:** The **difference** is 5.
- Teacher:** Excellent. **The difference between these two numbers is 5**. It isn't minus 5. **The difference between those numbers is 5**. OK? And if we put it on our number line, we can see it even more clearly. Minus 5 is here, zero is here and the **difference** is 5. OK? What is the next step, then? What is the next step? Clarion?
- Clarion:** We have to **halve the number**.
- Teacher:** OK. So, let's say that and repeat it. We're saying we have to **halve the number**. So, we have to **halve the number**.
- Class:** **Halve the number.**
- Teacher:** Excellent. So, can someone tell me what half of 5 is? What is half of 5? Kieran?
- Kieran:** 2.5.
- Teacher:** 2.5. Excellent
- Teacher:** Now that we've identified **half of the number**, we can look on our number line and what would my answer be here, then? Bredon?
- Bredon:** **Negative 2.5.**
- Teacher:** **Negative 2.5 or minus 2.5**. Well done. **Half way between 2 and 3**. So, it's **half way between 2 and 3**. Your turn.
- Class:** **Half way between 2 and 3.**
- Teacher:** Excellent. Well done. There you go. There's your answer.

Once the key vocabulary has been introduced within a unit, reviewing and embedding it is an ongoing process, as this video demonstrates. The teacher guides the discussion step-by-step so that the children can use the technical vocabulary of maths to explain what they are learning.

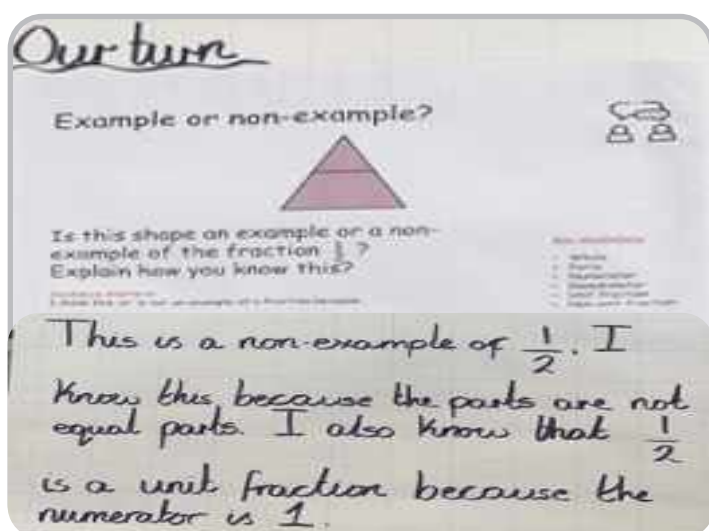
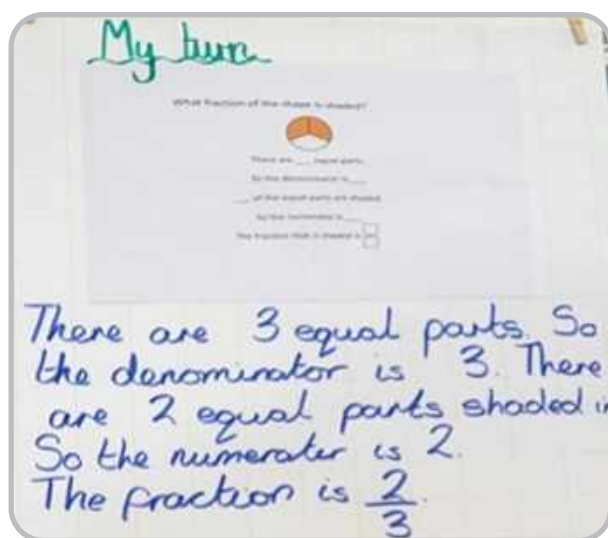
If you plan to use this video clip and related transcript for training, a useful question to ask is: **“How is the teacher using talk to help the children explain what they are doing and work out the maths correctly?”**

Use feedback to establish that it:

- provides opportunity for discussion, focusing on the use of explicit maths vocabulary, which helps everyone acquire the language;
- establishes the importance of asking questions clearly;
- requests answers in full mathematical sentences;
- points out key words, e.g. *difference*;
- gets all the children to say the key words together;
- gets all the children to say the key points together;
- gets the children to repeat each next step using the *my turn/your turn* technique;
- enables children to talk their way to understanding so that they know what to do and how to explain what they are doing.

My turn/your turn modelling in maths

In the images here from St Matthew’s, you can see that the teacher is using the *my turn/your turn* approach to help her Year 2 class talk their way to understanding fractions. At the *my turn* stage, the teacher models how to explain clearly what fraction of the shape is shaded.



The class then has a go at the *your turn* stage (referred to as *our turn* in the image here) so that the children are supported in expressing their understanding. In the *our turn* example here, the children have been provided with sentence stems that will enable them to explain coherently, alongside the technical vocabulary they need to express their understanding.

4. Make the learning visible

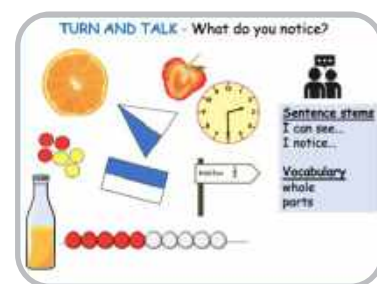
Throughout the **Imitation Stage**, the more the learning is made visible, the more the children are liable to understand and retain what they are learning – just as concrete visual aids help children grasp abstract concepts.

Tracey explains that once you have established the foundations of understanding through modelling and imitation, you can focus on specific skill objectives. For example:

To recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{4}$ ($\frac{1}{2}$) and $\frac{3}{4}$ of a length, shape, set of objects or quantity.

The more these abstract concepts can be represented visually by concrete everyday objects, the more quickly the children will be able to grasp the concepts, as the following steps illustrate.

1. To develop children's conceptual understanding of what a $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ look like, use images and talk to explore and recognise these unit fractions. This requires explicit teaching, for example modelling and teaching the link between $\frac{3}{6}$ and $\frac{5}{10}$ as pictured in the *Turn and Talk* image here. This illustrates how to support children's mathematical talk by sharing and modelling the use of key vocabulary and sentence stems – for example, *I can see...*, *I notice...* These sentence stems provide the key recyclable language patterns that will enable the children to explain their learning to themselves and others.

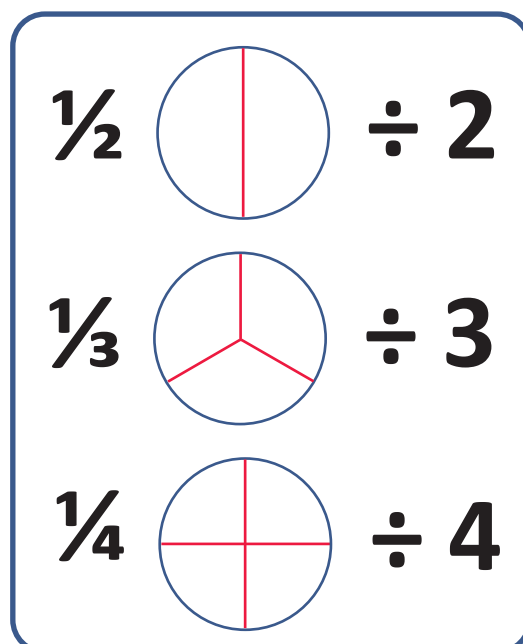


Going on a fraction walk would be an excellent way of extending the children's understanding of fractions, since it would deepen comprehension and sustain interest. Find and take pictures of fractions in the school environment. Can the children recognise and name the fractions? Can they find more examples?

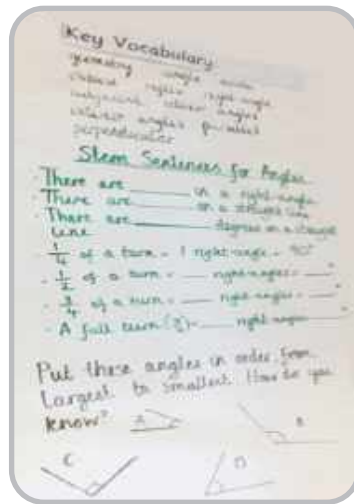
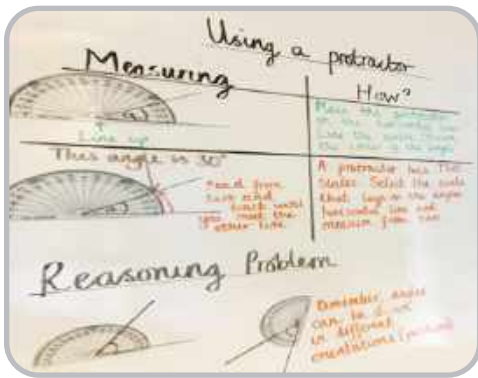
2. Focus on explicitly teaching children how to find a fraction of an amount. This is new learning, so it needs to be modelled carefully. The children will need their understanding of a *numerator* and a *denominator*, so review the text map.

Providing the same mathematical information in different formats, including pictorial ones, as illustrated here, is an excellent way of helping children comprehend: it links the fractions to the number of parts and reinforces understanding of the denominator, alongside showing how this can be represented when dividing numbers. The image bridges the gap between concrete and abstract.

It thus provides critical thinking opportunities and would lend itself to an open-ended booktalk-type discussion.



The more maths working walls or washing lines can be used to represent what is being learnt, the more powerful the learning will be, as illustrated by this working wall for Y5 geometry.



It encapsulates all the learning within the unit so far. It provides clear, simple guidance for using a protractor, and is highly visual so it is easy to refer to. It also provides key vocabulary for the unit, alongside stem sentences that will help the children express their understanding in precise mathematical language and undertake engaging challenges.

Co-construct the learning and how it is visually represented

One essential feature of maths working walls that scaffold learning effectively is that they are co-constructed with the class. If you look at the image here (which has also been recreated for clarity), the idea was that the *Post-it* notes were added to the working wall as the session developed so that the children were partners in the learning journey.

Finding half

The whole is

14

The whole is divided into

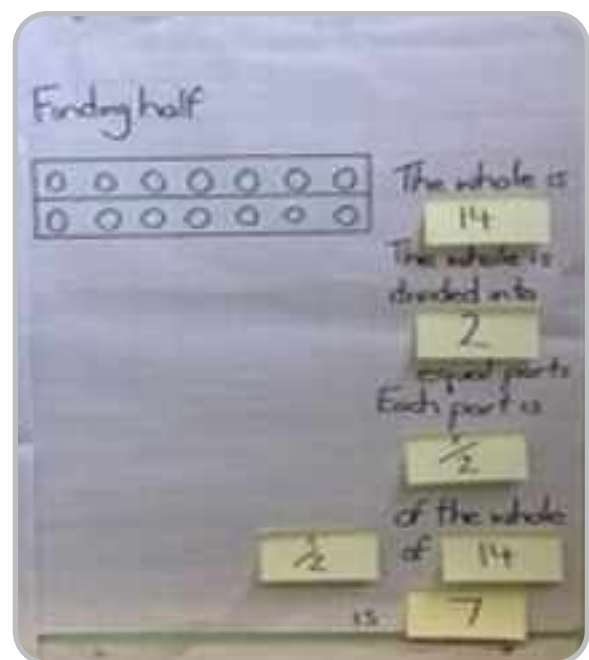
2

equal parts.

Each part is

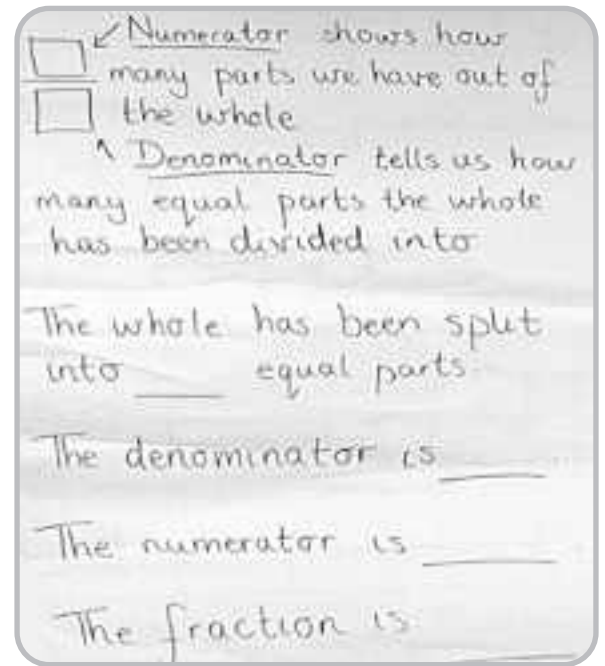
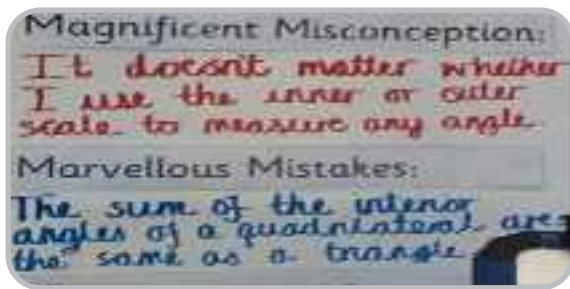
1/2 of the whole.

1/2 of **14** is **7**



The wall was accompanied by a flipchart (see the image on the right) reminding the children of the key language linked to the unit, alongside a cloze activity to help the children test their understanding of the terminology.

Maths working walls arm the children with the information they need to understand the work and enable them to counter misconceptions, as shown in the image below.

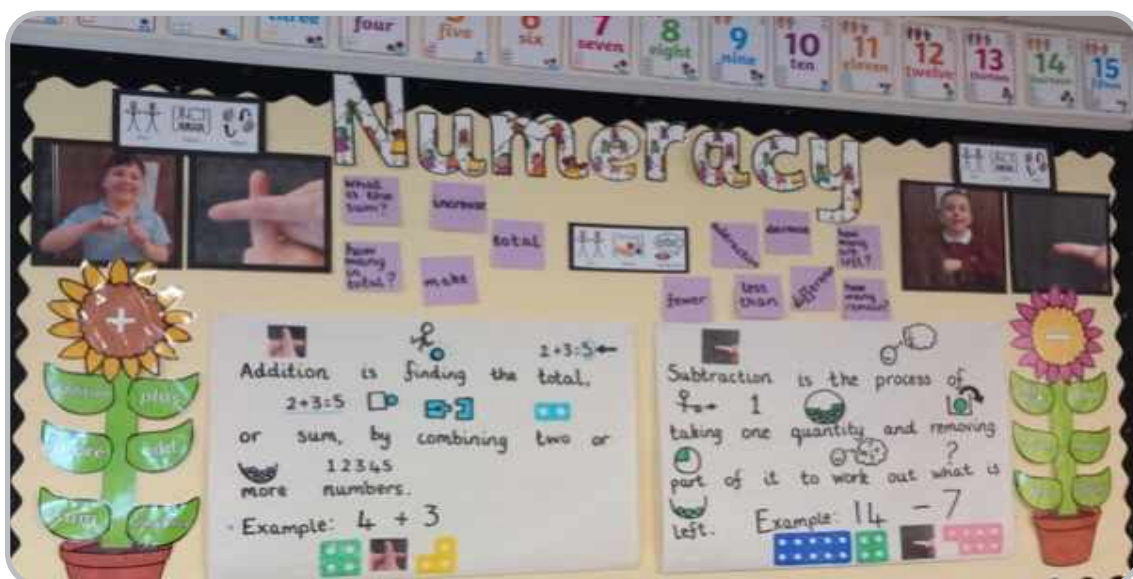


As the gap task at the end of the first TFW online maths session, delegates were asked to trial some of the ideas that had been in the first session and to send in examples of what seemed to make a difference. This resulted in some excellent feedback that underlines the importance not only of helping children understand key terminology but also of co-constructing that understanding and making the learning visible.

Dawn Osborne, from Larwood School in Hertfordshire (for children with social, emotional and behavioural difficulties), sent in the picture of the marvellous working wall below which she had just co-constructed with her class. She explained the difference it had made:

“We’ve always had a maths board, and the children who are able to read have always referred to it. This working wall is better because the children have the symbols they made and the definition. We have also incorporated a few widgit symbols which have helped our poorer readers to remember the definition.”

Dawn Osborne, Larwood School



Sarah Kimpton, Maths Lead at the same school, also co-constructed a working wall with her class following the online training and explained the difference it had made:

“We have been doing TfW since September and our pupils really enjoy literacy so much more and I enjoy teaching literacy so much more! I was so inspired by last week’s course that I changed my plans for maths for the rest of the week and started at the beginning with the vocabulary. My class are used to doing TfW so explaining what we were doing was easy, as they accepted it as normal.

To begin with, we did the work sheet of the key vocab and it became clear that they did not know all the words I had been using the week before – they were really struggling to get column addition. So, for the next lesson, we went through all the words – using an online dictionary to find meanings. The following day, we made up symbols for the four applications and the children took photos and made signs to put on the working wall – see image below. We have been using the signs in our lessons since then.



When it came to the column addition, I gave them some calculations and told them I was looking for the mistakes. They were stumped by this but did it anyway. From that exercise, I was able to see what I needed to teach and what their misconceptions were. I put them on the working wall. I broke down how to complete a column addition and how to partition a calculation. Within 2 days of practising these calculations, all of my pupils were confident in column addition! I had been trying for 2 weeks prior to teach this concept; following your course I was able to teach this in a few days! And the children enjoyed it. They are over the moon they can now do it! We had a showcase for the parents this week and it was wonderful to sit back and watch the pupils explain the maths working wall to their parents. One child, who really struggles with maths, got some scrap paper to show his mum he could actually do column addition!”

Sarah Kimpton, Maths Lead, Larwood School

5. Use text-mapping to internalise mathematical models

Imitation is essential for conceptual understanding. The text maps and actions for which the **Imitation Stage** of the TfW process is now famous are the perfect way to help children internalise and remember models of mathematical concepts and procedures because they help children understand concepts and their related language.

Tracey has found text maps to be a very effective support for maths:

“Previously, the key knowledge introduced within lessons was often soon forgotten and lost. Creating texts maps representing key mathematical knowledge has meant that the school now has a concrete way of getting children to internalise bodies of mathematical knowledge and embed it, just like they would do for a model text in English. It is something that can be used over and over again. Moreover, it also supports children’s ability to make connections between different areas of maths.”

Tracey Adams

Tracey illustrates how this applies to key concepts that Year 2 children have to understand, like knowing what fractions are, before beginning to unpick and then comprehend the objectives of the whole unit. Below is the text map she uses (with its transcription beside it) to help the children understand what fractions are.

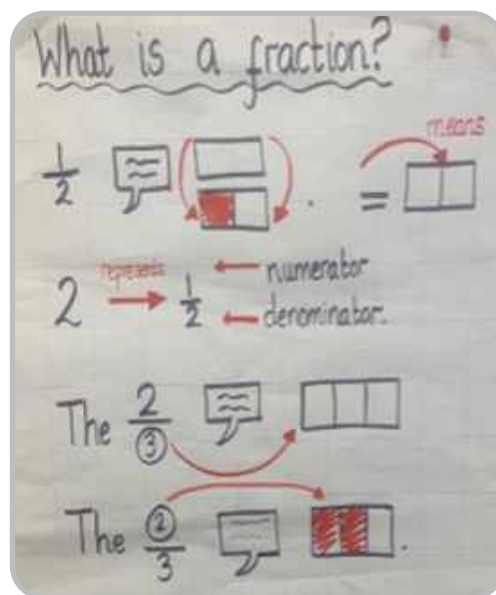
What is a fraction?

Fractions tell us how many equal parts of a whole we have. Equal means that all parts are the same.

To represent a fraction, we have the denominator and the numerator.

The denominator tells us how many equal parts the whole has been split into.

The numerator tells us how many we have.



In traditional TfW style, the children see the text map and imitate the language used alongside actions that help make the language meaningful. In this way, when they see or hear the language of fractions, they can understand it. The process helps build and embed their understanding so that they can use the language of fractions confidently to explain their learning.

Tracey also emphasises the need to revisit frequently the key learning points so that the children’s understanding becomes secure, as illustrated here. *Numerator* and *denominator* are very much the technical (Tier 3) language of fractions; without constant review, children’s understanding of such terms will fade.

Remember...

- 1** The numerator tells us how many parts of the whole we have.
- 4** The denominator tells us how many parts the whole has been broken into.

The more this *little and often* approach to revisiting key learning points becomes a part of everyday maths practice, the more the children will retain what they have learnt and develop their understanding.

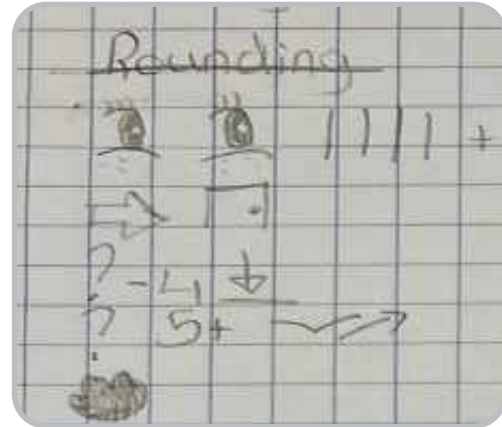
The image below (on the right) is a text map from Briar Hill which explains rounding to the nearest 10 and 100. On the left is the transcript describing what the symbols represent:

Rounding

Look in the column,
then go next door.

If it's 4 or less, then go to zero.

If it's 5 or more, fly high like a hero.



Text maps develop conceptual understanding by repeating in the mind how to go about doing things and by representing these stages visually. They help children think about what they have to do, and the repetition builds familiarity. They are also very supportive for EAL learners.

One delegate to online TfW maths training commented:

“I have been using text maps for the last couple of weeks. It is a SEND class with multiple personalised objectives so co-construction is tough. However, it has really helped me to better support and organise learning, enforce key messages to support staff and reduce anxiety for both children and staff around debugging misconceptions and challenges. Both children and staff have really taken to it.”

Video clip 6 of a Reception class at Anglesey Primary School shows how text maps and actions can help Reception children grasp very simple addition, with the abstract numbers being represented by the number of fingers the children hold up.

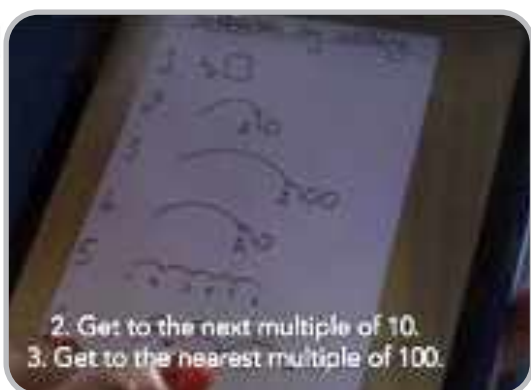
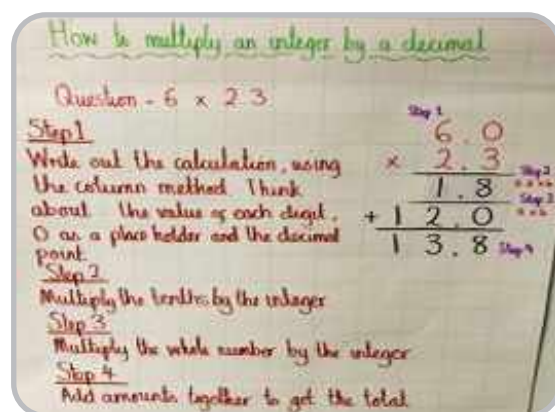
It's also worth watching this charming short film (**video clip 7**) from Katesgrove Primary School to see how text-mapping in maths helps children gain confidence in their ability to do maths.



6. Box-up the model's structure and co-construct its toolkit

Boxing-up is particularly useful in maths. Not only does it help children see the order in which something is done, but it can also – if the analytical boxing-up approach is used (see pages 28-31) – help children analyse the reasoning questions that sometimes make maths confusing. Simple text maps in maths are very much in a logical order, so they often have a dual function and operate as simple boxing-up, as **video clip 7** illustrates. The boy clearly knows the order of the steps he has to take, and he signals this with *First...*, *Next...* and *After that...* The text map has helped organise his thinking.

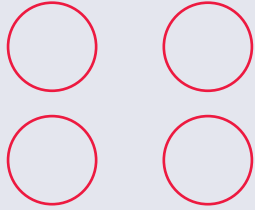
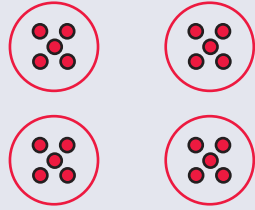
The flipchart pictured here shows the way in which Tracey models how to think through the steps of maths processes (procedural understanding) and writes the steps down in order. This is a simple form of boxing-up.



Video clip 8 illustrates the power of simple boxing-up in maths, as Year 3 teacher Ruth discovered. She was on a Tfw-across-the-curriculum project in Northampton and decided to introduce boxing-up into the way she taught subtraction by addition. As you watch the film, you will see what a difference it made. Boxing-up helps children clearly see each step they need to take. A good question to ask if you use this video is: ***“What aspects of the Tfw process is Ruth using, and how have they helped?”*** Use feedback to establish that:

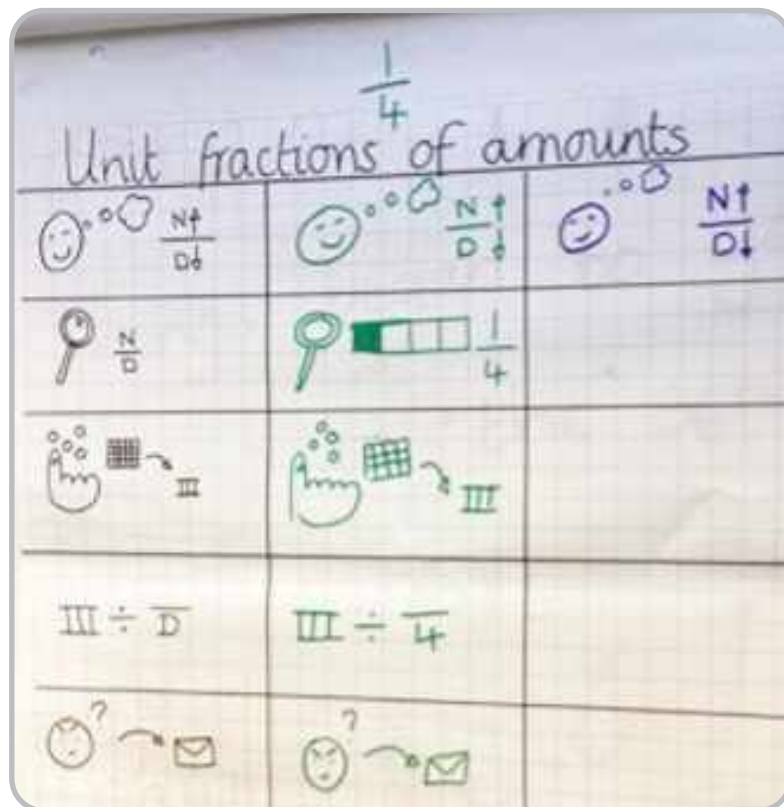
- the teacher boxed up the subtraction-by-addition process;
- she turned this into a text map;
- the children then learned orally with actions so they internalised it;
- the children then referred to the text map to support them;
- the visual nature of the text map helped them remember;
- they've internalised the steps so they can concentrate on the maths.

Boxing-up provides the perfect way of helping children structure the steps they will need to take to solve mathematical problems. Finding a fraction of an amount links together the conceptual ability to recognise a $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ and the procedural understanding of how to work out a fraction of an amount – the skills you need to apply to put your understanding about fractions into practice. As Tracey explains, this is the perfect opportunity to create a toolkit of what you need to do: since you have to do things in a specific order, as represented below, you can model them and box up the process. Tracey shows how boxing-up makes the procedural steps very clear alongside the conceptual understanding. To support the children's understanding, she also uses pictorial representations of what is happening which is reproduced on the next page.

Boxing-up the toolkit of what to do to find a fraction		
1. Read the question	<p>What is $\frac{1}{4}$ of 20?</p> <p>Think...</p> <ul style="list-style-type: none"> what is the whole? 20 what is the fraction? $\frac{1}{4}$ 	
2. Find a quarter	<p>To find $\frac{1}{4}$, I need to divide the whole (20) by 4. How do I know this? The denominator tells me how many parts I am sharing the whole into.</p>	
3. Divide by 4	<p>Now I know I am dividing by 4, I draw 4 circles. This is a pictorial representation of what is happening.</p>	
4. Share 20 between 4	<p>Share out 20 between the 4 circles. Look at one circle – 5. Focus on the fact that each circle has an equal amount. That amount is 5.</p>	

Again, this requires explicit teaching. The teacher models how the children can use their pictorial representation first to work out the unit fraction ($\frac{1}{4}$) and then extend to non-unit fractions like $\frac{3}{4}$ ($\frac{1}{2}$) and $\frac{3}{4}$; this thinking can then be extended to a $\frac{1}{3}$ and then $\frac{2}{3}$.

Briar Hill uses a text map-style toolkit in boxed-up steps to help children internalise the practical steps they need to take to find a fraction of something. This is illustrated by **video clip 9**, where Jordan Cross helps a Year 3 class internalise the process. As he talks the children through the process, he uses the text map of the boxed-up steps (pictured here) to help the children see exactly where they are within the process. If you are using this clip for CPD, you might want to ask your audience to think about this question while watching: **“Why does internalising all the steps of the process in boxed-up order help the children apply their understanding?”**



Video clip 10, where Allán Crozier is also teaching a Year 3 class, further demonstrates how, once the children have internalised the text map, you can use the boxed-up process to deepen their understanding of fractions. The image from the clip shows how important it is to make the learning visible to support the children’s understanding. If you are using this clip for CPD, you might want to ask your audience to consider this question while watching: **“How is the teacher helping the children deepen their understanding and application of fractions?”**

You might then want to flipchart all the points and ask everyone to watch the clip again and see if there is anything to add.

Analytical boxing-up helps solve reasoning questions

In online TfW maths training, we asked the question: **“What makes the reasoning questions in maths hard?”** Not surprisingly, responses poured in, suggesting that the reasoning questions are indeed proving challenging. The essence of the feedback was this:

- cognitive overload – decoding language/question before thinking about the maths needed.

This is where analytical boxing-up comes in. In around 2010, when running a *TfW-across-the-curriculum* project with teachers and advisers from a range of schools in Brighton, Zeb Friedman, who was both a teacher and a maths adviser in Brighton at that time, had been an unwilling attendant. She hadn’t felt it would be any use for maths. But she suddenly

realised, as I explained boxing-up, that this was exactly what she was looking for to solve the problem of how to tackle the wordy maths problems that, in today's KS2 SATs, are called *reasoning questions*. This table shows how she turned the boxing-up explanation for text-based subjects (left column) into boxing-up how to solve maths problems (right column).

How boxing-up an explanation text in English led to boxing-up in maths – making it easier to understand the steps needed to solve maths problems

Boxing-up explanation in text-based subjects	Boxing-up maths problems
Introduce what is being explained	What is the question asking me? What information do I have?
Middle section (explain the process): <i>This leads to this because of... and, therefore,...</i>	What maths will I be using?
	Working out/calculations
Conclusion and check work	Check answer and presentation

Adapted from work of Zeb Friedman, Maths Consultant and Maths Teacher in Brighton

Zeb went away and trialled her ideas and came up with the maths boxing-up mat reproduced here. The idea is that the children work in pairs following the steps outlined on the mat. First, they discuss what the question is actually asking and, once that has been agreed, establish what information they have been given that will help answer it. After that, they move on to the blue box and discuss what maths they will use to answer the question. Next, they do whatever working out/calculations are necessary and, finally, check their answer. Initially this is done in pairs, but over time the children internalise the process and automatically follow the same procedure when working on their own.

What is the question asking me?

What information do I already have?

What calculations/working out do I need to do?

What maths will I be using?

How can I check that my answer is correct?

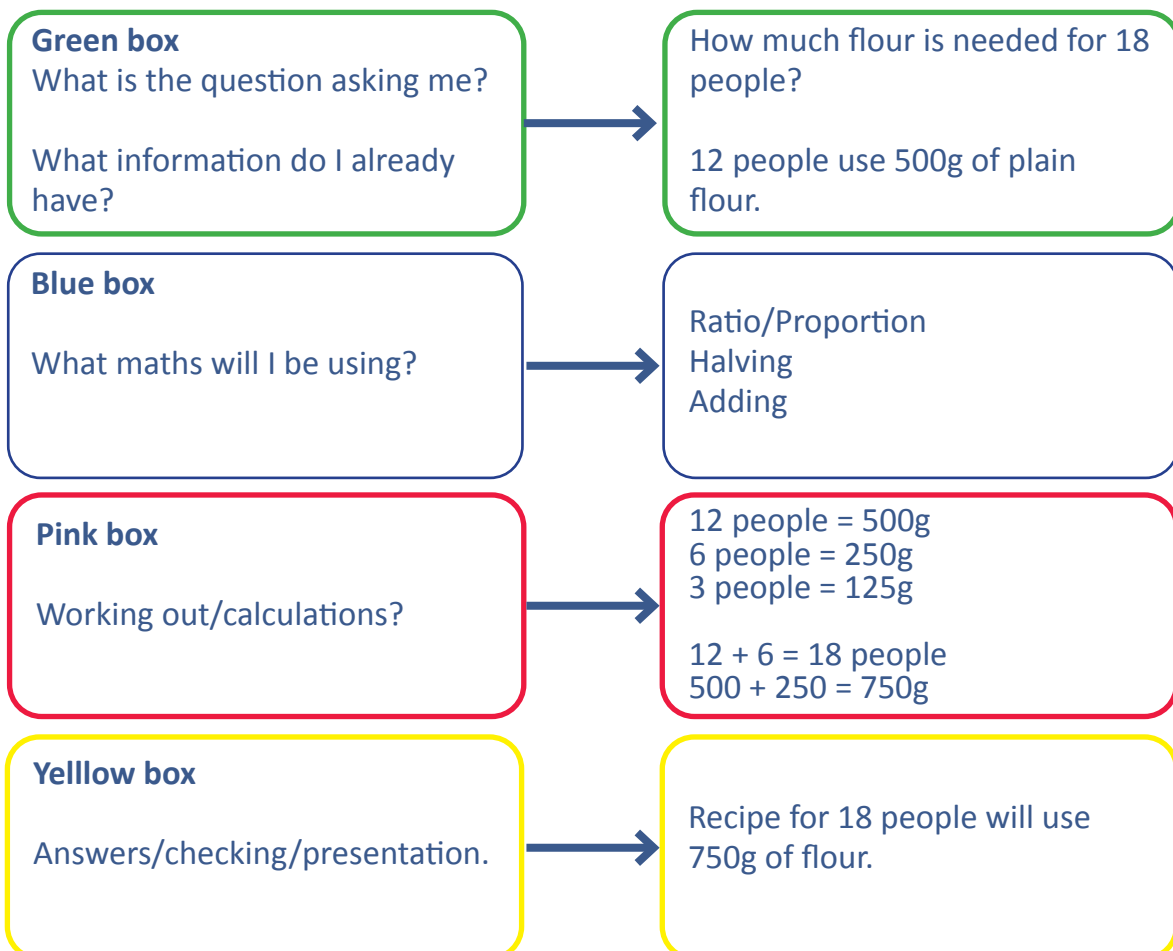
At the same time, I had introduced *analytical boxing-up in maths* to the Head of maths at Villiers High School in Southall as part of a TfW project there. She then trialled the approach with her department and found that it really made a difference, as these comments from three teachers in the maths department illustrate.

“More excellent work from my Years 8s. Topic: Volume of cylinders: find missing length or radius. Very good student discussion. Students said boxing-up is the exact method they were looking for: better understanding and breaking down of a worded question.”

“Please see attached TfW work my Year 9s did. I am now just asking them to use the TfW boxing-up method for any worded problem and this is what they produce. I don’t now need to spend extra time prompting them. We are getting there.”


“TfW with Year 8 class. In future lessons, I will be asking them to use the boxing-up method when answering questions that are more worded. I feel that it was successful and it was easier to address misconceptions.”

The process had been introduced to the students using questions like this: **If a recipe for 12 people uses 500g of plain flour, how much flour is need for 18 people?**



When I visited the school in June 2023, I saw this process in action. The teacher presented the problem (pictured at the top of the image here) and also provided the students with the boxed-up answers to each section but in the wrong order (pictured at the bottom half of the image). The students' task was to discuss the problem and decide what was the correct order for the boxing-up and then write a brief explanation of the order selected, as illustrated. I asked a range of students whether this approach was helping and the replies were resoundingly positive, accompanied by smiles because it was giving them the confidence to tackle the problems.

Q) Malia is flying a kite on a 20 m long string. The string is at an angle of 35° to the horizontal. Malia is holding the kite 1.1 m above the ground. Find the vertical height of the kite above the ground in metres.



Below are the steps involved to answer the question. Put the steps in the correct order and explain in words what each step is for.

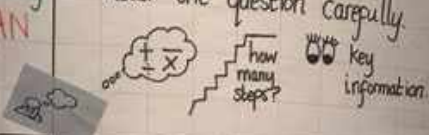




STEPS	Correct order	EXPLANATION
$11.47 + 1.1 = 12.57\text{m}$	5	To find the final answer
$\sin 35 = \frac{y}{20}$	3	It's setting up the equation
Hypotenuse = 20m and Opposite side = y Angle = 35°	1	It's telling use the key information that we need.
SOH CAH TOA	2	It's telling use the methods we need to pick from.
$20 \times \sin 35 = y$	4	It's rearranging the question to figure out the missing means.



The picture here shows the class in pairs discussing the problem before deciding how to fill in the boxes to explain the rationale of the order they have chosen. Zeb Friedman, who first developed this approach, told me that the pairing up of children to discuss the problems really helped. If the children need more support, the teacher can quickly identify those children who cannot find a route through the wording and help them.

Here you can see how St Matthew's has developed its own simple version of analytical boxing-up in text-mapping style, so that the children automatically go through the right planning process to solve problems. The images are a very useful addition, as they remind the children of the importance of the initial thinking stage when they plan how to approach solving the problem. It also includes the suggestion that the children draw images to help them picture the problem.

How can we problem solve in Maths?

Beginning PLAN IT	Read the question carefully. 
Middle DO IT	Work it out - Draw it...  - step by step  - Write it down 
End CHECK IT	Check your answer - Is it reasonable? - Calculate the inverse 

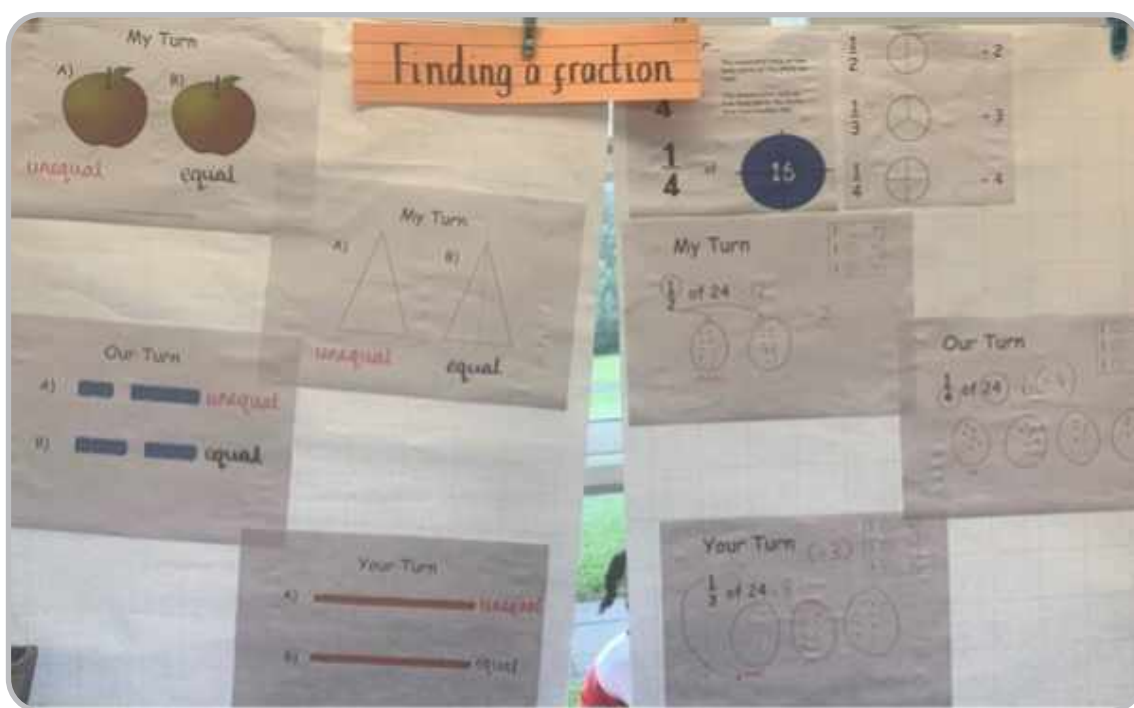
Chapter 2: The Innovation Stage

Once the **Imitation Stage** has firmly underpinned understanding of whatever the unit is focusing on, it is time to move on to the **Innovation Stage**.

The only meaningful test of the efficacy of what has been taught at the **Imitation Stage**, and the way it has been taught, is whether it provides effective foundations for the children to be able to innovate successfully during the **Innovation Stage** – albeit with continuing scaffolded support where necessary.

1. Demonstrate innovation through shared calculations

Tracey explained that during the **Innovation Stage**, the Year 2 children develop their conceptual understanding of what a fraction is and how to find a fraction of an amount. The teacher models how to apply what they have learnt about fractions through shared calculations related to the toolkit, just as shared writing supports understanding in English. As a result, the children are ready to have a go at a number of questions, using the toolkit as a support alongside the working wall/washing line that visually represents what they have been learning:



The washing line pictured above was developed during the **Imitation Stage**, but it now underpins the development of that learning at the **Innovation Stage**. It is a vital tool to support children's application because it provides visual support for their understanding.

The **Innovation Stage** also provides the following assessment opportunities.

- Who is able to find a fraction of an amount?
- Can they use a pictorial representation to support them?
- Can they identify the answer and explain how they came to it?

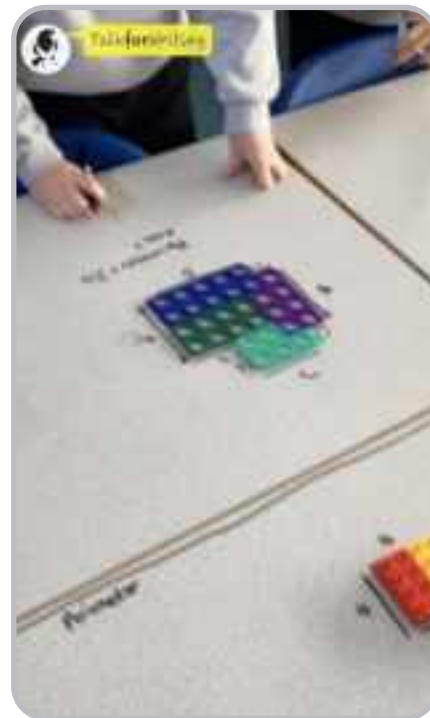
This enables the teacher to know which children will need more support before they can move on to the **Independent Application Stage**. If they attempt that stage without this underpinning understanding, this can only result in confusion and a loss of confidence – the *I can't do maths* syndrome will set in.

2. Talk through the innovated process

Now, let's have a look at how Year 6 is getting on with understanding fractions. The context of this film ([video clip 11](#)) is listed at the side of this screenshot from the clip.



1. *The children are shown how to apply the same approach to a different question.*
2. *In small groups they apply the same approach to a different question.*
3. *Then they create their own questions for other groups to solve.*



When watching this clip, you can see how concrete resources like *Numicon* help the children secure their knowledge of fractions by helping them visualise the problem they are solving; this deepens their understanding. They can manipulate the resources, which opens discussion and encourages them to talk about the logic underpinning what they are doing so that they talk their way to understanding. This understanding is strengthened because, at the final stage, they have to create questions for other groups to answer, which encourages further focused discussion. If you are planning to use this clip as part of CPD, a useful question to ask is: ***“How are the concrete resources helping the children secure their knowledge of fractions?”***

3. Use analytical boxing-up to structure the innovation

Before moving on to the **Independent Application Stage**, let's have a look at what difference analytical boxing-up made to the secondary students' ability to tackle challenging problems.

When you look at the example below, you can see that this student has successfully analysed the challenging question and then come up with two methods for solving the problem, both of which have arrived at the same answer, which gives him confidence that his answer is right. He also suggests comparing his answer with those of his peers.

Innovation Stage – example of how a Y9 student used this analytical approach

Below is a shape. Some of the corners are right angles others are not. All dimensions are in cm. Calculate the area of the shape.

Shape A
Shape B

What is the question?
What is the area of the shape?

What Maths will I be using?

- Geometry.
- Formulae.
- Algebra (simplifies expressions)
- Addition and subtraction.

What calculations/working out do I need?
 $x(2+3x) = 2x + 3x^2$
 $22.5x - 7.5x^2 + 2x + 3x^2 = 24.5x - 4.5x^2$
 $24.5x - 4.5x^2$
 Area of shape A
 $\frac{(10-x+5) \times 3x}{2} = \frac{15-x}{2} \times 3x$
 $\frac{45x - 3x^2}{2} = 22.5x - 1.5x^2$ → Area of shape B
 $2x \times x = 2x^2$ → Area of shape M
 $\frac{(5+x+10) \times 3x}{2} = \frac{15+x}{2} \times 3x = \frac{45x + 3x^2}{2} = 22.5x + 1.5x^2$
 $22.5x + 1.5x^2 + 2x = 24.5x + 1.5x^2$
 Full area of the shape

How can I check my answer to see if it's correct?
 I can check the answers of my peers to make sure that I got the correct answer.

Method 2 for checking my answer
 Both answers the same: this means my work is correct.

Maths department, Villiers High School

4. Write down understanding to embed retention

Tracey has found that getting the children to write down their understanding coherently is a powerful way of helping them retain their learning. If the children can write down the thinking behind getting the answers right clearly and mathematically accurately, it means that they have fully understood. If they can explain it to themselves and to others, they will be able to retain this understanding.

At the same time as being introduced to analytical boxing-up, the maths department at Villiers was also influenced by Tracey's advice.

On the next page you can see how this has been put into practice alongside, in this example, simple boxing-up. As illustrated earlier on **page 30**, all the steps to getting the answer have been provided. The task is to box up the steps in the right order and then explain in words the reasoning behind each step.

Task: Put these steps in order to make a solution and write the final answer. Give reasons in words for each of your steps

$$g = \frac{6.67 \times 10^{-11} m}{r^2}$$

Earth	Jupiter
$m = 5.98 \times 10^{24}$	$m = 1.90 \times 10^{27}$
$r = 6.378 \times 10^6$	$r = 7.149 \times 10^7$

Find $g_{\text{Earth}} : g_{\text{Jupiter}}$ in the form 1: n

$(6.378 \times 10^6)^2 = 4.068 \times 10^{13}$

$6.67 \times 10^{-11} \times 1.90 \times 10^{27} = 1.27 \times 10^{17}$

$(7.149 \times 10^7)^2 = 5.111 \times 10^{15}$

$\frac{3.99 \times 10^{14}}{4.068 \times 10^{13}} = 9.81$

$\frac{1.27 \times 10^{17}}{5.111 \times 10^{15}} = 24.8$

$6.67 \times 10^{-11} \times 5.98 \times 10^{24} = 3.99 \times 10^{14}$

1:2.53

$\frac{24.8}{9.81} = 2.53$

Here is one student's response (it has also been transcribed alongside the original):

Solution:

Reason in words

Step1: $6.67 \times 10^{-11} \times 5.98 \times 10^{24} = 3.99 \times 10^{14}$	Calculate the value of the numerator in the formula of earth.
Step2: $(6.378 \times 10^6)^2 = 4.068 \times 10^{13}$	Calculate r^2 in the denominator.
Step3: $\frac{3.99 \times 10^{14}}{4.068 \times 10^{13}} = 9.81$	Divide the equation to find gravity on earth.
Step4: $6.67 \times 10^{-11} \times 1.90 \times 10^{27} = 1.27 \times 10^{17}$	Calculate the value of the numerator in the formula of Jupiter.
Step5: $(7.149 \times 10^7)^2 = 5.111 \times 10^{15}$	Calculate the value of the denominator in Jupiter.
Step6: $\frac{1.27 \times 10^{17}}{5.111 \times 10^{15}} = 24.8$	Divide equation to find gravity on Jupiter.
Step7: $\frac{24.8}{9.81} = 2.53$	Calculate 'n' from the ratio of 1:n.
Step8: 1:2.53	Measure written in the form 1:n.

1. Calculate the value of the numerator in the formula of earth.
2. Calculate r^2 in the denominator.
3. Divide the equation to find gravity on earth.
4. Calculate the value of the numerator in the formula of Jupiter.
5. Calculate the value of the denominator in Jupiter.
6. Divide equation to find gravity on Jupiter.
7. Calculate 'n' from the ratio of 1:n.
8. The answer written in the form 1:n.

The full power of this approach will become clear in the **Independent Application Stage** of the TFW process.

Chapter 3: The Independent Application Stage

1. The Hot task

This final stage of the process provides the children with the opportunity to show what they can now do independently. This is known as **The Hot task** or **Show what you know/The Challenge task/The Exit ticket**, when the children apply what they have learnt independently to a different question relating to similar content, or to a different problem using the method that has just been learnt. Depending on the unit, preparation for the **Independent Application Stage** will vary. In some situations, feedback from the **Innovation Stage** may mean that key skills need embedding before the hot task can begin.

If some children have not made sufficient progress to be able to tackle the final stage independently, their work should still be scaffolded so that they can make progress. It is pointless making a child sit through something that may last several days in order for them to demonstrate that they can't do what you both knew they couldn't do at the outset.

So, let's have a look at the learning journeys of the fraction units we have already seen.

Here is Tracey's **Challenge task** for her Year 2 group. As you can see, the first activity supports the children in talking their way to understanding: the children are challenged to explain what they have learnt and how they know it is right.

Challenge task

$\frac{1}{3}$ of 12 is the same as $12 \div 3$.

I agree because...

I disagree because...

Exit ticket

1. To find $\frac{1}{2}$, I divide by _____.

2. To find $\frac{1}{4}$, I divide by _____.

They then tackle their **Exit ticket** to demonstrate that they can apply their understanding.

The final challenge is a reasoning question including the instruction to **Explain why**. This is taxing, since both the fractions and the whole sums of money are varied. There are several logical pitfalls that might lead to a fall at this fence.

Questions like this help the teacher know that the children have really understood because they have to explain using the appropriate technical vocabulary, rather than just follow a process.

Who has more? Explain why.

Rosie: I have $\frac{1}{4}$ of £8

Whitney: I have $\frac{1}{2}$ of £6

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Tracey emphasises that now is also the time to explore this key question: **“How flexible is the children’s knowledge around fractions of an amount, outside of the procedure?”** Space needs to be created in the teaching journey to:

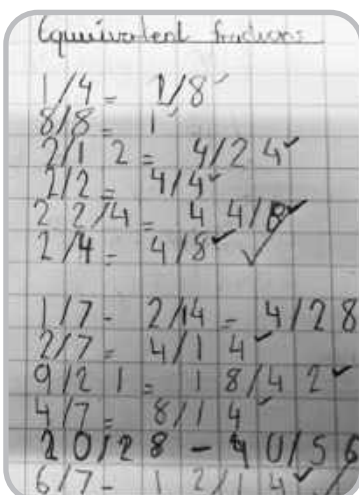
- retrieve key knowledge;
- apply what they know to problems.

The use of talk partners and whole-class discussions enables the teacher to identify understanding and address misconceptions. What responses are you looking for to show conceptual and procedural understanding?

The cold-to-hot task gives you a whole-class overview of your children’s understanding of the key learning objectives of the unit. The children can see from the cold task how much they have progressed, and the teacher can see how the children have progressed over time. Significantly, this process enables the teacher to pull together all the learning and see which concepts are secure and which need to be revisited in a new unit to establish secure basic understanding.

“Once the understanding is embedded, it’s a case of how am I going to give the children different opportunities to apply it? How does the maths we’ve just done link to the next bit of maths we’re doing and how do I support the children so that they can see that connection? The teacher has to decide what needs retrieving and what needs embedding to make the learning secure. The cold-to-hot process greatly enables this.”


Tracey Adams



By the time the children reach Year 5 in Briar Hill, the fractions **Hot task** might look like the activity pictured here. The children are given the fractions you can see on the left of the image. Their task is to write equivalent fractions next to them.

You can see that this child has understood that if you double one side (the denominator) you have to double the other (the numerator) – this is the key.

Shade $\frac{1}{4}$ of this shape.



$\frac{1}{4}$ of 12 = 3 ✓
 $12 \div 4 = 3$ ✓
 $12 \times 1 = 12$ ✓

In addition, the children were given questions like these that represent the same information but in a different way. This is also more challenging because it uses irregular shapes, not just squares or circles, which are often used to represent fractions. Such questions test whether they can apply their understanding.

“This shows deeper understanding – it’s a SATs style question to see whether the Year 5 children have independent understanding of what $\frac{1}{4}$ of 12 means. They need to be able to put into words the thinking process that is going on in their head to get to the right answer. This then enables them not only to answer the reasoning problem but to express it mathematically in a range of ways – they can articulate their thinking.”

Nick Warren

The **Hot task** continued with a range of activities to show that the children can articulate their thinking coherently.

A common denominator is when the denominator in 2 or more fractions are common, or the same.


Example: $\frac{5}{20}$ $\frac{4}{20}$ ✓

$\frac{1}{4}$ $\frac{1}{5}$ The denominators are not the same. The denominators are not common. ✓

$\frac{5}{20}$ $\frac{4}{20}$ The denominators are the same. The denominators are common. ✓

Complete an explanation next time.

When ordering decimals, you need to look at what column the digits are in. Like on the 1st question, it was between 2.03 and 2.04, 2.033 would have went 3 divisions after 2.03 because 2.033 is 0.003 after 2.03.



A = $5.3 \times 10 = 53$ ✓
 B = $6.4 \times 10 = 64$ ✓
 C = $5.3 \times 100 = 530$ ✓
 D = $6.4 \times 100 = 640$ ✓
 E = $5.3 \times 1000 = 5300$ ✓
 F = $6.4 \times 1000 = 6400$ ✓
 Well done!

When multiplying and dividing by 10, 100, and 1000 you need to move the numbers either left or right. When multiplying, you move the digits left and by the amount of place holders (0's) in the number you are multiplying by. It is just the same for dividing, however, you move to the right. Remember, the decimal point stays aligned, in the same place) and never moves. ← × • = → ✓

Great explanation

2. Embed what they have learnt

Writing down the rationale behind your understanding is an excellent way of embedding learning. In the activity on the previous page, you can see that the teacher, when marking the work, has requested that the child: **“Complete an explanation next time”**. The child clearly sees the logic of this and has immediately written an explanation that shows he understands the process he has been using.

As a further extension to the initial teaching, and as a chance to deepen understanding, Tracey asked the children to write a set of instructions around **five months** after the teaching took place to explain the objective in detail (in this example, multiply an integer by a number with up to 2 decimal places).

How to multiply a decimal by an integer

Maths is a wonderful subject, fascinating bright young minds. However, it can scratch and scabble some heads. Like many other challenging subjects' concepts, decimals takes a little bit of understanding. This text will assist you in solving the trickiest of problems: multiplying an integer by a decimal.

You will need:

a pencil

a ruler

paper

a calculation

Method:

- *First, create a problem you can solve, e.g. 7.92×8 . Then, set out your calculation. Ensuring every digit is in its right column, will result in the correct figures, which is why it is one of many high priorities in maths, e.g.*

$$\begin{array}{r} 7.92 \times \\ \quad 8 \\ \hline \hline \end{array}$$
- *Next, multiply your last digit (hundredths) with the multiplier. It is essential to exchange the excess because without it your final answer would be incorrect, e.g.*

$$\begin{array}{r} 7.92 \times \\ \quad 8 \\ \hline \quad 6 \\ \hline \quad 1 \end{array}$$
- *After that, multiply the next digit to the left (tenths) with the multiplier. Do not forget to add the exchanged number.*
- *Finally, multiply the first digit (ones) with the multiplier. Do not forget the decimal point. Your final answer would be much larger than it is meant to be if you were to forget it.*

In Tracey's words:

"The child's writing shows that their understanding of both the procedure and concept is extremely secure and flexible, as it can be applied within a completely different context."

Pie Corbett commented:

"A window onto genuine learning – great assessment. Plus being fun! Expecting children to sequentially explain processes will aid deepening of understanding."

Conclusion: Building in progression

Maths results in both St Matthew's and Briar Hill are outstanding, and all the more so when considering how challenging the intakes of both schools are. So it is worth considering what has helped create such results.

Tracey emphasised how the support given to the teachers to help them teach maths effectively underpinned this success:

"We've really unpicked the quality of the teaching of what's delivered in the classroom and supported the teachers to help them understand the subject knowledge behind what they are teaching. You could just pick up a maths scheme and, if you are not careful, you don't even look at the National Curriculum – you just follow the scheme – but this will not result in powerful teaching and learning. You help all the teachers to see how the units are broken down into concepts and procedures, and the rationale behind the order of each unit, and the order of the units so they understand how to develop progression.

The subject knowledge behind the teaching is crucial. What we have done here is unpick good pedagogy that works whether it's maths or geography or any other subject. They are used to the cold-to-hot process; they are used to explicit modelling; they are used to turn and talk, to think/pair/share: all of these things they are very familiar within all their lessons. The basic maths planning is all in place – the only bit of planning the teacher has to do is to identify the productive struggle – the aspects of the unit that will cause the biggest stumbling blocks for their class. If I'm moving into a unit on fractions, which are the hardest bits of teaching that I'm really going to have to think about how to teach? And what models or images will I need to support the children so they can tune into the hardest parts? This preparation is essential – you have to have thought through how to overcome any stumbling blocks otherwise there will be no progression. Once you've identified the areas that will cause the most difficulty, you can seek out help from your subject leader so that you can go to the lesson with confidence and overcome any problems. In such ways, teachers who do not feel confident teaching maths become confident and can teach their classes effectively so that the children become confident mathematicians.

An essential part of this is the way the key TfW strategies support maths because they emphasise the children's ability to talk their way to understanding. Without these opportunities to talk through their understanding of the concepts and procedures they are working on, all deep learning is lost. You can't show a connection in maths when teaching unless you talk it through – you model it through but you must also give the children the opportunity to talk their way through as well...

We map out the progression that shows where you think the children should be. We've thought very clearly about where we think the children should be as mathematicians by Year 6 – our end point. Then we create progression documents to show how the children can get there. In addition, we recognise each child comes to us with knowledge. We've got to create opportunities for them to show what they already know and build on it."

Here, Nick explains how the TfW process has helped Briar Hill build in progression:

“Briar Hill offers a high-quality mathematics education which aims to provide a foundation for understanding the world while developing an appreciation of the beauty and power of mathematics alongside a sense of enjoyment and curiosity about the subject.

This is achieved through building on prior knowledge through repeated units of work that progressively develop higher understanding. Aiming high while scaffolding down through talk partners is an embedded and sustaining strategy which allows children to understand concepts and internalise them. The curriculum’s design is imperative to the success of the school.

Progression at Briar Hill is based on the TfW process, so it begins with the cold task. This helps assess prior knowledge while identifying which areas need further focus and support alongside any whole-class interventions that might be needed. This is not normally the case at Briar Hill, as assessment for learning takes place throughout every lesson through ‘live marking’ and on-the-spot interventions. The TfW process has been vital to success in mathematics. In particular, the following elements of the TfW process have proved particularly effective:

- ***developing vocabulary – using the Isabel Beck technique;***
- ***providing models, examples and non-examples – analysing primary/secondary sources;***
- ***using sentence stems to develop the tune of maths;***
- ***text-mapping key learning points/knowledge/concepts;***
- ***boxing-up the order in which to do the maths;***
- ***investigating a problem (boxing-up mathematical problems).***

Briar Hill’s inclusive approach aims for the vast majority of children to access learning in their age-appropriate class. This enables them to build on previous learning in each new step of their learning journey. As a result of building essential maths knowledge and skills cumulatively, children become confident, resilient mathematicians, demonstrating conceptual and procedural fluency, with the ability to reason mathematically and solve problems.”

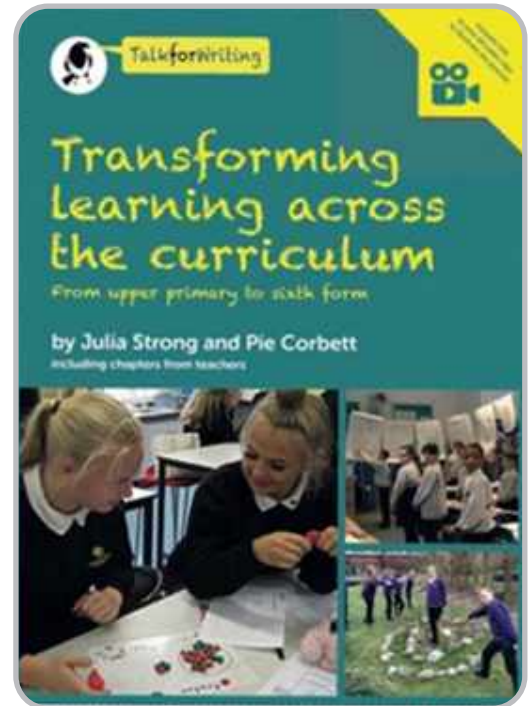
For schools already using the TfW approach in English, it makes excellent sense to integrate all the key features of TfW into how you teach maths as well as all the other subjects, so that the children have one unifying powerful approach to learning across the curriculum.

If you want to know more about how the underpinning pedagogy of Tfw supports learning across the curriculum, see:

- ***How Talk for Writing supports Science*** (online resource) by Julia Strong and Pie Corbett – available spring 2024
- ***Transforming Learning across the Curriculum*** by Julia Strong and Pie Corbett – available from <https://shop.talk4writing.com/products/transforming-learning-across-the-curriculum>

“Who’d have thought Talk for Writing could make such a difference to maths, PE and science? Packed with riches, the powerful Tfw approach is applied in this book to the entire curriculum by the experts Pie and Julia, with outstanding results. You will find everything you need to be transformative in developing students’ language acquisition and confidence in writing, no matter what the subject. Detailed rationale, links with cognitive science, endless practical examples across the upper primary and secondary curriculum and exemplary chapters written by teachers: get started!”

Shirley Clarke, Formative Assessment Expert



Appendix 1: Why St Matthew's and Briar Hill use TfW to support maths

In 2009, when Paulette Osborne became Headteacher, St Matthew's SATs results were, in her words, dire: 34% combined. She decided what the school needed was Talk for Writing. By 2015, the combined result was 97%, despite a Pupil Premium average of 88%. The assessment system is more challenging now, but the combined score remains equally high. **In 2022, 97% of KS2 pupils achieved the expected level in maths** (26% above the national average of 71%), with **31% achieving greater depth** (national average 22%).

Tracey Adams (former Maths Lead at St Matthew's, now Head at Christ Church C of E Primary School) explained that integrating the pedagogy behind TfW into the way they teach maths has had a great impact on the children's ability to think and work mathematically, as it provided a chance to encounter mathematical concepts and procedures through a successful strategy that the children were already very familiar with.

Prior to this joined-up way of thinking, teachers did not have a clear understanding of what children were able to do within a specific area of maths. The teachers lacked a clear way to assess what children were bringing to the table and what they were able to do after a unit of work had been taught. TfW's cold-to-hot-task structure helped to bring real clarity to the data teachers had before beginning to teach and gave them the knowledge needed to develop connections as the teaching continued, alongside building the children's mathematical skills and hence boosting confidence.



There were also key bits of mathematical knowledge that the teachers wanted to embed into the children's long-term memory, but previously the key knowledge introduced within lessons was somehow soon forgotten and lost. Creating text maps representing key mathematical knowledge has meant that the school now has a concrete way of getting children to internalise bodies of mathematical knowledge and to embed it, just like they would do for a model text in English. It is something that can be used over and over again. Moreover, it also supports the children's ability to make connections between different areas of maths.

In addition, teachers had seen the way that unpicking particular aspects of writing, through boxing-up and co-constructing toolkits, had had a powerful impact on writing, and they wanted to apply this to maths. Using this approach gives the children a chance to see a teacher model a particular procedure, so that they can see the thinking behind it, just as modelling in writing helps them see the thinking that lies behind selecting which words to use. This enabled teachers in maths to unpick what a mathematical problem was and to show the thinking. In such a way, teachers could clearly identify the steps to be taken alongside the related tools to be used to solve mathematical problems and achieve an accurate answer.

Tracey summed up by saying:

“The big concepts which teachers are trying to explore with children can be greatly enhanced by embedding them within the pedagogy of Talk for Writing. It supports the teacher in breaking down the learning into small chunks of teaching, with quality modelling and models at the core of developing children’s understanding and application. It enables both the teacher and child to co-construct how to box up structure and create toolkits of key ingredients, which can then lead to independent application. It is a brilliant tool for promoting quality dialogic talk within mathematics and, with thought, can be used as a vehicle for writing in maths, as children’s subject knowledge is so secure, they can create their own texts from a place of real confidence...”

Briar Hill Primary School has been using the TfW approach across the curriculum for many years. Not only has this caused results to improve dramatically but it has also provided a stable foundation for pupil development and the culture of the school. Previously, Briar Hill used a number of different schemes and new initiatives which often caused confusion. Since the TfW approach has been used right across the curriculum, results in maths have outstandingly improved. Despite an EAL intake and Pupil Premium of over 50%, mathematic results in KS2 have averaged 90% since 2017; in 2022, 43% achieved greater depth (nearly twice the national average of 22%). In 2015, when the school didn’t use TfW, their maths results were well below the national average. In 2022, Briar Hill won the Times Educational Supplement (TES) Primary School of the Year and their Ofsted inspection graded them as Outstanding in every category.



Nick Warren, Year 6 teacher and maths lead at Briar Hill, explained that having the core principles and pedagogy behind TfW embedded across the school means that the children have already encountered key strategies such as hook tasks, boxing-up and cold-to-hot tasks, so they are used to units leading up to the independent application of what they have learned. This ensures that the children’s learning focuses on thinking and working mathematically, as it provides a chance to encounter mathematical concepts and procedures through a successful strategy that the children are already familiar with.

Previously, when various teaching styles and strategies were used, alongside a range of worksheets, the children’s specific needs often weren’t being met. TfW’s cold-to-hot process has enabled staff to have a clear understanding of formative baseline assessment and also provides a great summative assessment tool. This allows teachers to focus on developing the areas that most need support while building on the children’s prior knowledge. Providing the appropriate small steps to develop the learning and build connections ensures the children’s mathematical skills and confidence develop hand in hand.

TfW techniques such as shared writes, which in maths often become shared calculations, co-constructing toolkits and boxing-up the order in which to do things, mean that every child is able to achieve; learning is deep and sustainable as the toolkit can be referred back to and helps address any misconceptions. Not only do toolkits or shared calculations build confidence, but they also help to ensure that learning is built on something that has already been sufficiently mastered. The children talk their way to understanding, so they

can reason about concepts and make connections, which in turn develops conceptual and procedural fluency. This enables the teachers to unpick what a mathematical problem is and demonstrate the thinking. In such a way, teachers can clearly show how to box up the steps to be taken alongside the related tools to be used to solve mathematical problems and achieve an accurate answer.

Consequently, children leave Briar Hill with the ability to reason mathematically and solve problems effectively, as the school's results illustrate.

Appendix 2: Vocabulary progression for maths from Briar Hill

Year 1 (includes general vocabulary and phrases introduced in EYFS and further developed throughout EYFS and Year 1 curriculum that are also useful in maths)

Number and place value	Addition and subtraction	Multiplication and division	Measure	Geometry: position/direction	Geometry: shape	Fractions	General/problem solving
<ul style="list-style-type: none"> number zero, one two, three – twenty and beyond none count (on/up/to/from/down) before, after more, less many, few, fewer, least, fewest, smallest, greater, lesser equal to, the same as odd, even pair units, ones, tens ten more/less digit numeral figure/s compare in order/a different order size value between, halfway between above, below 	<ul style="list-style-type: none"> number bonds, number line add, +, more, plus, make, sum, total, altogether inverse double, near double half, halve equals, =, is the same as difference, difference between How many more to make...? How many more is... than...? How much more is...? subtract, -, take away, minus How many fewer is... than...? How much less is...? addend subtrahend minuend sum difference 	<ul style="list-style-type: none"> odd, even count in twos, threes, fives count in tens (forwards from/backwards from) How many times? lots of, groups of once, twice, three times, five times multiple of times, multiply, multiply by repeated addition array, row, column double, halve share, share equally group in pairs, threes, fives, etc. equal groups of... divide, divided by, left, left over factor multiplicand multiplier product quotient divisor dividend 	<ul style="list-style-type: none"> full, half-full, empty holds container weight, weighs, balances heavy, heavier, heaviest; light, lighter, lightest scales time days of the week: Monday, etc. seasons: Spring, etc. day, week, month, year, weekend birthday, holiday morning, afternoon, evening, night, midnight bedtime, dinnertime, playtime today, yesterday, tomorrow before, after next, last now, soon, early, late quick, quicker, quickest; quickly, fast, faster, fastest; slow, slower, slowest, slowly old, older, oldest; new, newer, newest takes longer, takes less time hour, o'clock, half past clock, watch, hands How long ago? How long will it be to...? How long will it take to...? How often? always, never, sometimes, often, usually once, twice first, second, third, etc. estimate, close to, about the same as, just under, just over too many, too few, not enough, enough length, width, height, depth long, longer, longest; short, shorter, shortest; tall, taller, tallest; high, higher, highest low, wide, narrow, deep, shallow, thick, thin far, near, close metre, ruler, metre stick money, coin, penny, pence, pound, price, cost, buy, sell, spend, spent, pay, change, dear/er, costs more, costs less, cheaper, costs the same as How much? How many? total 	<ul style="list-style-type: none"> position over, under, underneath, above below, top, bottom, side on, in, outside, inside around, in front, behind front, back before, after beside, next to, opposite apart between, middle, edge, centre corner direction journey left, right, up down, forwards, backwards, sideways across close, far, near along, through to, from, towards, away from movement slide, roll, turn, whole turn, half turn stretch, bend 	<ul style="list-style-type: none"> group, sort cube, cuboid, cone, pyramid, sphere, cylinder, circle, triangle, square shape flat, curved, straight, round hollow, solid corner (point, pointed) face, side, edge make, build, draw 	<ul style="list-style-type: none"> whole equal parts, four equal parts one half, two halves a quarter, two quarters 	<ul style="list-style-type: none"> listen, join in say, think, imagine, remember start from, start with, start at look at, point to put, place, fit arrange, rearrange change, change over, split, separate carry on, continue, repeat, and what comes next? find, choose, collect, use, make, build tell me, describe, pick out, talk about, explain, show me read, write, record, trace, copy, complete, finish, end fill in, shade, colour, tick, cross, draw, draw a line between, join, join up, ring, arrow count, work out, answer cost count, work out, answer, check, same number/s, different number/s, missing number/s number facts, number line, number track, number square, number cards abacus, counters, cubes, blocks, rods, die, dice, dominoes, pegs, peg board same way, different way, best way, another way in order, in a different order not all, every, each

Year 2							
Number and place value	Measure	Geometry: position/direction	Data/statistics	Fractions	Geometry: properties of shape	General/problem solving	
<ul style="list-style-type: none"> numbers to one hundred hundreds partition recombine hundred, more/less 	<ul style="list-style-type: none"> quarter past, quarter to m/km, g/kg, ml/l temperature, degrees 	<ul style="list-style-type: none"> rotation clockwise, anticlockwise straight line ninety-degree turn, right angle 	<ul style="list-style-type: none"> count, tally, sort vote graph, block graph, pictogram represent group, set, list, table label, title most popular, most common, least popular, least common 	<ul style="list-style-type: none"> three quarters, one third, a third equivalent, equivalence 	<ul style="list-style-type: none"> size bigger, larger, smaller symmetrical, line of symmetry fold match mirror line, reflection pattern, repeating pattern 	<ul style="list-style-type: none"> predict describe the pattern, describe the rule find, find all..., find different... investigate 	
Year 3							
Number and place value	Measure	Geometry: position/direction	Geometry (properties of shape)	Fractions	Addition and subtraction	Multiplication and division	Data/statistics
<ul style="list-style-type: none"> numbers to one thousand 	<ul style="list-style-type: none"> leap year twelve-hour clock/ twenty-four-hour clock Roman numerals I-XIII 	<ul style="list-style-type: none"> greater/less than ninety degrees orientation (same orientation, different orientation) 	<ul style="list-style-type: none"> horizontal, perpendicular and parallel lines 	<ul style="list-style-type: none"> numerator, denominator unit fraction, non-unit fraction compare and order tenths 	<ul style="list-style-type: none"> column addition and subtraction 	<ul style="list-style-type: none"> product multiples of four, eight, fifty and one hundred 	<ul style="list-style-type: none"> chart, bar chart, frequency table, Carroll diagram, Venn diagram axis, axes diagram
Year 4							
Data/statistics	Measure	Geometry: position/direction	Geometry (properties of shape)	Fractions and decimals	Multiplication and division	Number and place value	
<ul style="list-style-type: none"> continuous data line graph 	<ul style="list-style-type: none"> convert 	<ul style="list-style-type: none"> coordinates translation quadrant X-axis Y-axis perimeter and area 	<ul style="list-style-type: none"> quadrilaterals triangles right angle, acute and obtuse angles 	<ul style="list-style-type: none"> equivalent decimals and fractions 	<ul style="list-style-type: none"> multiplication facts (up to 12x12) division facts inverse derive 	<ul style="list-style-type: none"> tenths, hundreds, decimal (places) round (to nearest) thousand more/less than negative integers count through zero Roman numerals (I-C) 	
Year 5							
Number and place value	Addition and subtraction	Measure	Multiplication and division	Geometry: position/direction	Geometry: properties of shape	Fractions, decimals and percentages	
<ul style="list-style-type: none"> powers of 10 	<ul style="list-style-type: none"> efficient written method 	<ul style="list-style-type: none"> volume imperial units, metric units 	<ul style="list-style-type: none"> factor pairs composite numbers, prime number, prime factors, square number, cubed number formal written method 	<ul style="list-style-type: none"> reflex angle dimensions 	<ul style="list-style-type: none"> regular and irregular polygons 	<ul style="list-style-type: none"> proper fractions, improper fractions, mixed numbers percentage half, quarter, fifth, two fifths, four fifths ratio, proportion 	
Year 6							
Number and place value	Addition and subtraction	Fractions, decimals and percentages	Multiplication and division	Data/statistics	Geometry: position/direction	Geometry: properties of shape	Algebra
<ul style="list-style-type: none"> numbers to ten million 	<ul style="list-style-type: none"> order of operations 	<ul style="list-style-type: none"> degree of accuracy simplify 	<ul style="list-style-type: none"> order of operations common factors, common multiples 	<ul style="list-style-type: none"> construct mean pie chart 	<ul style="list-style-type: none"> four quadrants (for coordinates) 	<ul style="list-style-type: none"> vertically opposite (angles) circumference, radius, diameter 	<ul style="list-style-type: none"> linear number sequence substitute variables symbol known values

Appendix 3: Tracey Adams' overview of planning a Year 2 fractions unit

Planning for any unit will always begin with having a clear purpose. Tracey Adams explains that the Year 2 National Curriculum objectives for fractions provide a clear focus for the unit. Children should be taught to:

- recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity;
- write simple fractions – for example, $\frac{1}{2}$ of 6 = 3 – and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$.

But, as always, building on prior knowledge to facilitate progression is key, so this is dependent on whether the children, by the end of Year 1, are able to:

- recognise, find and name a half as one of two equal parts of an object, shape or quantity;
- recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.

St Matthew's approach to teaching maths ensures that teachers have a clear understanding of the progression around the key mathematical concepts they are developing so that by the time the children enter Year 2 their underpinning understanding of fractions is secure.

This underlying substantive knowledge for fractions begins in Reception when the children explore the concept of *parts* and *whole*. The language and concrete understanding is developed through modelling, exploration, story and concrete manipulation.

The picture here shows how the concrete manipulation of counters plus squares divided into quarters can help young children conceptualise what a *quarter* means.

The language of *part* and *whole* is crucial to exploring the concept of a half and a quarter, and there is also a need to explicitly teach additional vocabulary, which would include:

- *denominator;*
- *numerator;*
- *quantity;*
- *shape;*
- *equal;*
- *half;*
- *quarter.*



Appendix 4: Nick Warren's overview of planning a Year 6 unit on fractions

Block 1 – Year 6 – Fractions Unit 1

Planning Stage	Begin with lesson involving counting in fractional steps, which revises some core knowledge about fractions, including equivalence and terminology. It also reinforces the fact that fractions can be greater than one.
Cold task to establish baseline	
1. Imitation Stage	Children consolidate their understanding about finding fractions of quantities. Procedural variation is used to find fractions of amounts. For example: $\frac{3}{4}$ of £5.00 is calculated by finding half of £5.00 and then finding a quarter of £5.00; finding three quarters then involves combining these amounts. As well as solving problems involving finding fractions of a whole, children engage in problems where the whole is unknown and must be derived from a fractional part.
Developing fraction notations through prior knowledge	
2. Innovation Stage	Knowledge about equivalent fractions is then developed. Learning focuses on the concept (as the number of parts increases/decreases we need more/less of them to cover the same area) and the procedure (if you multiply/divide the numerator you must multiply/divide the denominator).
Support understanding of structure, talk the process of innovation and use models.	
3. Independent Application Stage	After consolidating knowledge about equivalent fractions, children learn to recognise when fractions are not in their simplest form. They use their understanding of common factors (from Year 5) to simplify fractions, including dividing the numerator and denominator by the highest common factor to express a fraction in its simplest form. The unit concludes with work on comparing and ordering fractions. Children compare fractions using representations, using reasoning (when numerators or denominators are the same) and without representations.
Apply what they have learned independently	
Lessons and evidence from books	<ol style="list-style-type: none"> 1. Review from gaps – e.g. counting in sixths and eighths. 2. Finding fractions of quantities. 3. Equivalent fractions. 4. Simplifying fractions. 5. Comparing and ordering fractions (two lessons). 6. Comparing fractions using reasoning.